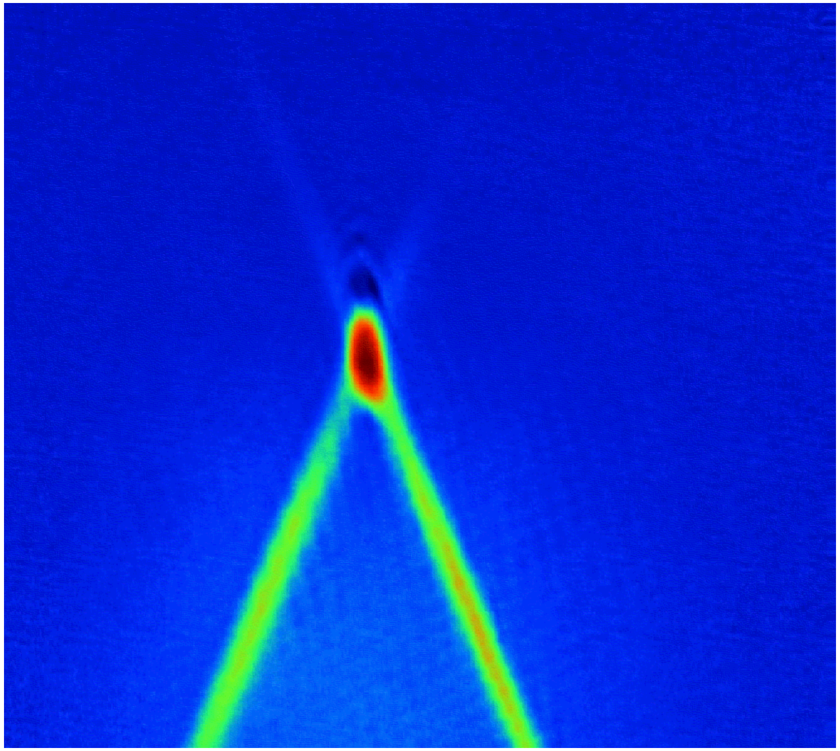
Tuning interactions in ultracold Bose and Fermi gases



C. Salomon
Laboratoire Kastler Brossel, Ecole Normale Supérieure, Paris
http://www.lkb.ens.fr/recherche/atfroids/welcome

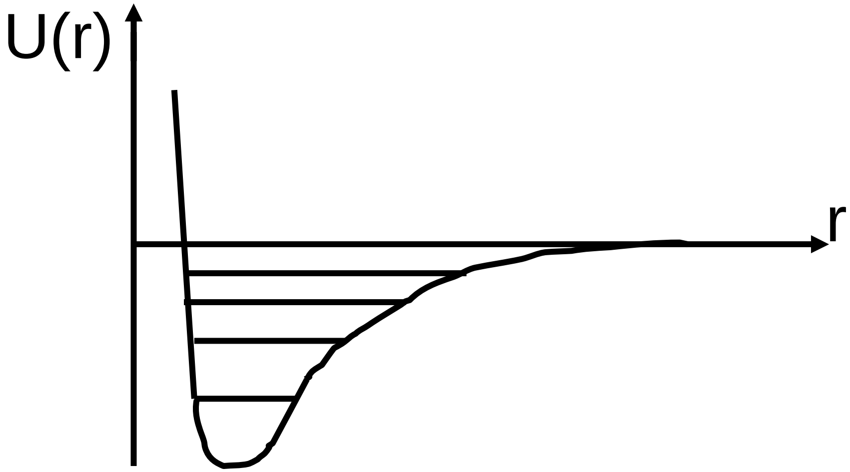
Heraklion, July 2007

Outline of the 2 lectures

- Bose gases: bright and grey Solitons in Bose-Einstein Condensates
  Example of application of Gross-Pitaevskii equation

- Fermi gases: exploration of BEC-BCS crossover
  Superfluidity in Fermi gases

Atom-atom interactions



At low temperature,
only s wave collisions

$$\psi(\vec{r}) = e^{i\vec{k}.\vec{r}} - \frac{a}{r} e^{ikr}$$

$$a = -\lim_{k \to 0} \frac{\tan \delta_0(k)}{k}$$

The magnitude and sign of ***a*** depend sensitively on the detailed shape of long range potential
Importance of position of last bound state
Using a DC magnetic field, one can modify the potential.

a: scattering length
|*a*|~1 to 100 nm

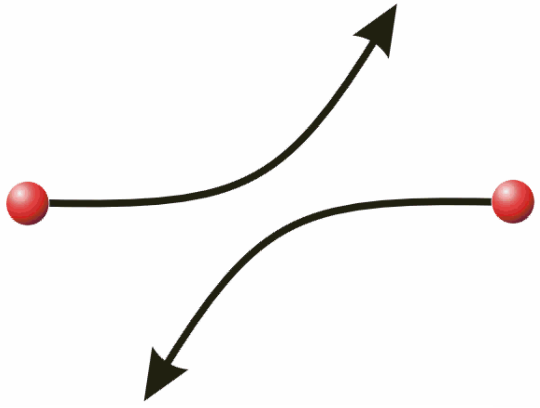
$$V(\vec{r}_1 - \vec{r}_2) = \frac{4\pi\hbar^2 a}{m} \delta(\vec{r}_1 - \vec{r}_2)$$

a can be adjusted from $-\infty$ to $+\infty$
Feshbach resonance

a > 0 : effective repulsive interaction
a < 0 : effective attractive interaction

Mean field in Bose-Einstein Condensate

- At very low temperatures one parameter sufficient to describe interaction: the scattering length a



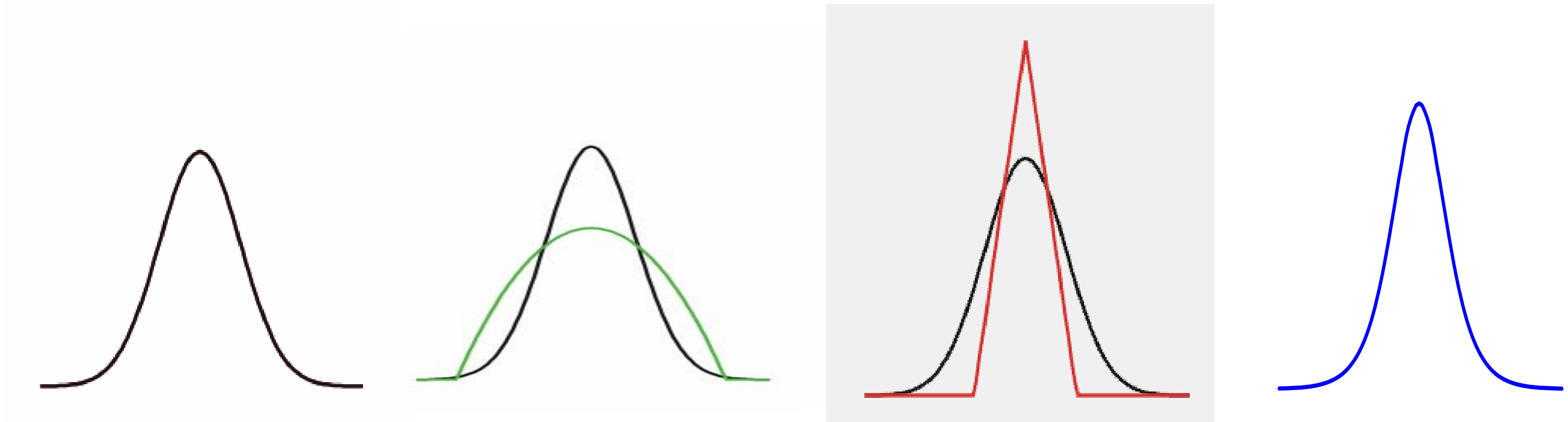
- Scattering cross section: $\sigma = 4\pi a^2$
  (non identical particles)
- Mean field of a gas with density n

$$U = \frac{4\pi\hbar^2 n a}{m}$$

- Example: BEC in dependence of mean field

Ideal gas: a=0 Repulsive: a>0 Attractive: a<0

3D gas 1D gas



Gaussian Parabola Collapse (for $N>N_{crit}$) Soliton

Sandro Stringari lecture

BEC in harmonic trap

$$V_{ext} = \frac{1}{2} m\omega_{ho}^2 r^2$$

Non interacting ground state

$$n(r) \propto \exp(-r^2 / a_{ho}^2)$$

Gaussian with width $a_{ho} = \sqrt{\frac{\hbar}{m\omega_{ho}}}$

depends on $\hbar$

Role of interactions

Using a_{ho} and $\hbar\omega_{ho}$ as units of lengths and energy, and $\tilde{\Psi} = N^{-1/2} a_{ho}^{-3/2} \Psi$

normalized to 1

GP equation becomes

$$[-\tilde{\nabla}^2 + \tilde{r}^2 + 8\pi(Na/a_{ho})\tilde{\Psi}^2(\tilde{r})]\tilde{\Psi}(\tilde{r}) = 2\tilde{\mu}\tilde{\Psi}(\tilde{r})$$

dimensionless Thomas-Fermi parameter

If $Na / a_{ho} << 1$ **Non interacting** ground state

If $Na / a_{ho} >> 1$ **Thomas-Fermi** limit (a>0)

What happens for negative a ?

In 3D: condensate collapse if

$$N\,|a|/a_{ho} > 1$$

Ruprecht et al. PRA 51 (1995)
Bradley et al. (1997)
Roberts et al. (2001)

$$E_{GP}[\psi] = \int d^3\vec{r}\left(\frac{\hbar^2}{2m}|\nabla\psi(\vec{r})|^2\right) + \frac{gN}{2}|\psi(\vec{r})|^4 + [\frac{1}{2}m\omega_z{}^2 z^2 + \frac{1}{2}m\omega_\rho{}^2(x^2+y^2)]\,|\psi(\vec{r})|^2$$

$$g = 4\pi\hbar^2 a/m < 0$$

The interaction energy shrinks the cloud, increasing further the interaction energy and overcomes the kinetic energy (KE) term

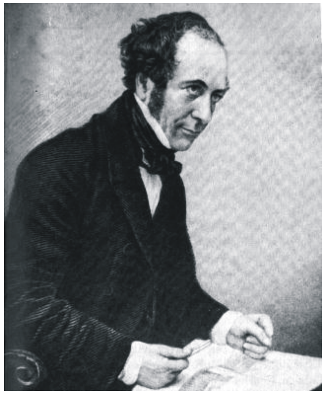
If $N\,|a|/a_{ho} < 1$ Condensate is stable but with very few atoms (<100 !)

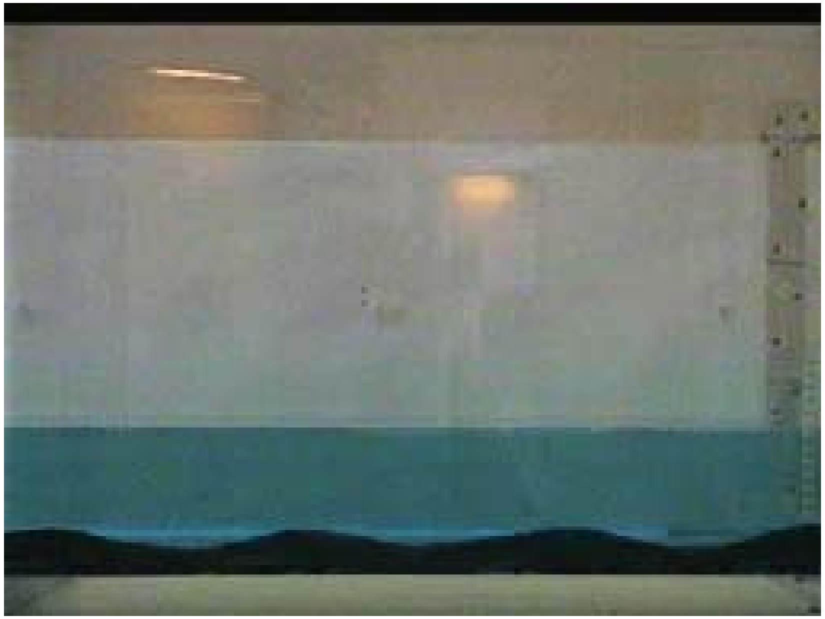
In 1 D: possibilty to have a cancellation between KE and interaction energy: $N_0 \dfrac{g_{1D}}{l}$ $\dfrac{\hbar^2}{ml^2}$

SOLITON

Soliton

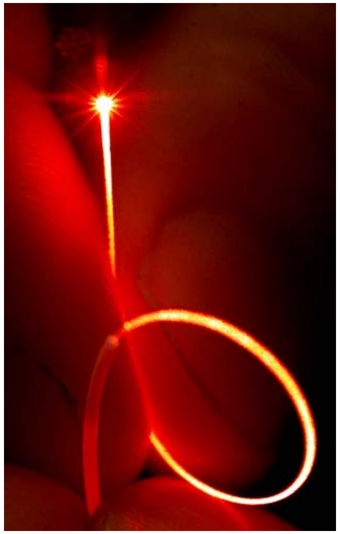
- Dispersion counterbalanced by non linear interaction
- Discovered 1834 by Scott Russell in water





- Used in optical fibers for telecommunication

  Non linear Schroedinger equation



Appears in many fields of Science and Technology !

Formation of Matter Wave Solitons

For positive a, (87 Rb, Na), a soliton is an excited state of a BEC which can be excitedby engineering a special phase and amplitude upon a BEC
Hannover, NIST, Harvard, JILA

Another elegant method : band soliton (Heidelberg, M. Oberthaler)
Shift the sign of the mass
In a lattice, the effective mass can be negative !

For negative a, a soliton is a stable self-interacting quantum system which propagates over large distances without attenuation nor dispersion

Matter wave soliton

Assumption: start with a BEC such that $\hbar\omega_\perp \gg N_0 |g| |\phi|^2$ No collapse

Condensate wavefunction $\phi(x,y,z) = \psi(z)\chi(x)\chi(y)$

with harmonic oscillator ground state wave function along x and y

Gross-Pitaevskii energy functional

$$E_{GP}(\psi) = N_0 \int dz \left[\frac{\hbar^2}{2m} \left| \frac{d\psi}{dz} \right|^2 + \frac{1}{2} m\omega_z^2 z^2 |\psi(z)|^2 + \frac{1}{2} N_0 g_{1D} |\psi(z)|^4 \right]$$

with

$$g_{1D} = g \frac{m\omega_\perp}{2\pi\hbar} = 2a(\hbar\omega_\perp)$$

$$g_{1D} \leq 0$$

Soliton (2)

What happens if one turns down slowly ω_z ?

Well known bright soliton solution for $\omega_z = 0$

$$\psi(z) = \frac{1}{(2l)^{1/2}} \frac{1}{\cosh(z/l)}$$

Spatial size of soliton: $$l = -\frac{2\hbar^2}{N_0 m g_{1D}}$$

Trade-off between minimization of kinetic energy $\frac{\hbar^2}{ml^2}$

and interaction energy per particle: $N_0 \frac{g_{1D}}{l}$

Chemical potential: $$\mu = -\frac{1}{8} N_0^2 \frac{m g_{1D}^2}{\hbar^2}$$

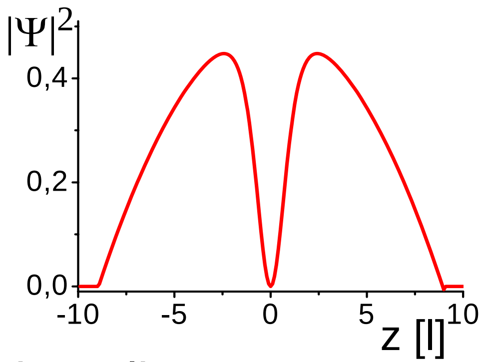
With, of course, $\mu \leq \hbar\omega_\perp$

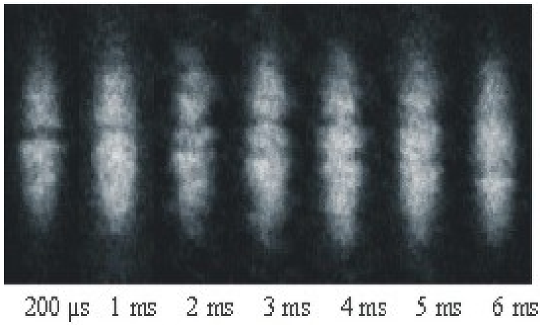
Matter Wave Soliton

- Dispersion of matter wave $E = \hbar^2 k^2 / 2m$
- Non linear interaction due to mean field

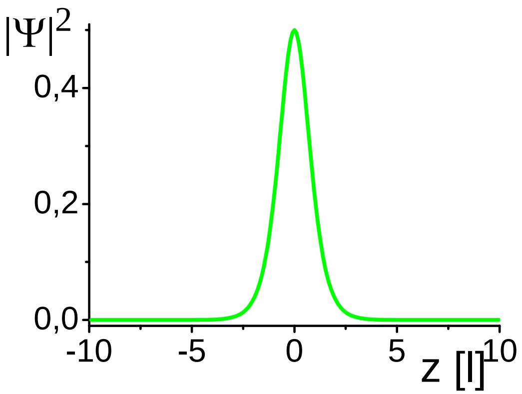
1D GPE: $$\mu\psi(z) = -\frac{\hbar^2}{2m}\frac{d^2\psi(z)}{dz^2} + (\frac{1}{2}m\omega_z^2 z^2 + Ng_{1D}|\psi(z)|^2)\,\psi(z)\ ;\ g_{1D} = 2a(\hbar\omega_{rad})$$

- Dark solitons created in BEC in Hannover, NIST, in 2000, JILA, Harvard, a>0





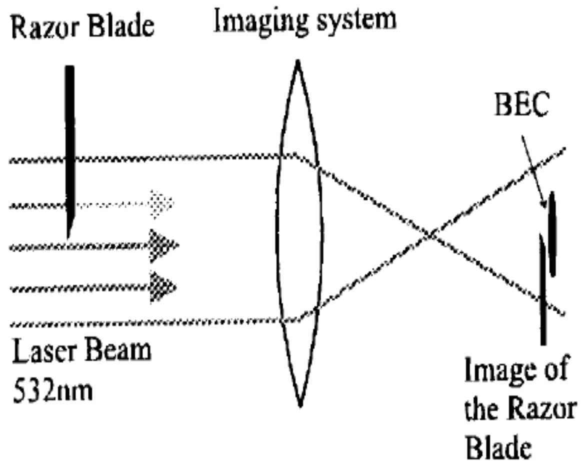
- Bright Solitons



$$\Psi(z) = \frac{1}{(2l)^{1/2}\cosh(z/l)}$$ With $l = -\sigma_{ho}^2 / Na$

$$N_{\max} = -\sqrt{2}\,\sigma_{HO} / a$$ For 1D condition

Phase imprinting on BEC



Apply π phase shift to half of the condensate wavefunction with a far detuned Laser beam

Time t_p short compared to h/μ

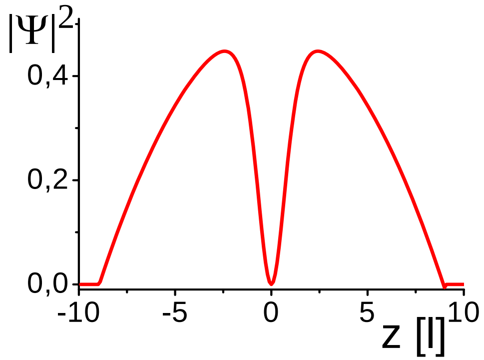
50 Watt/mm^2

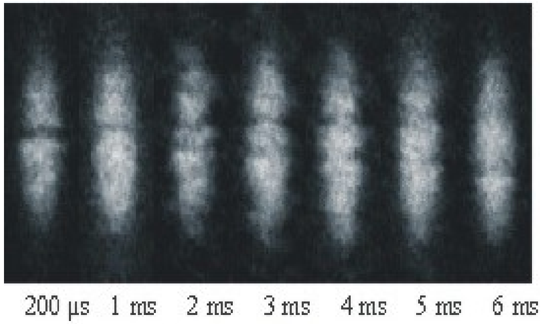
Lambda = 532 nm

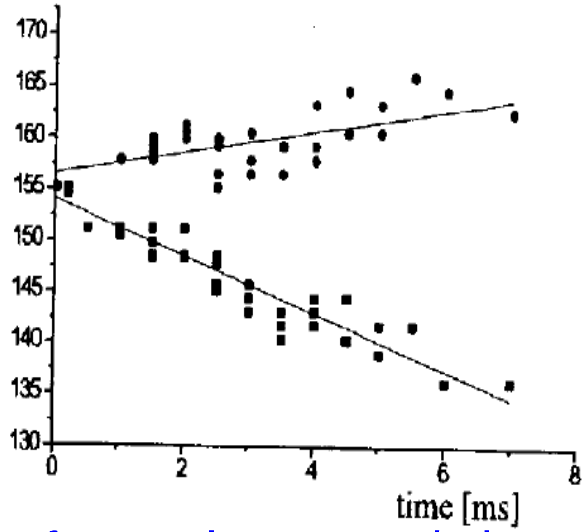
U= 5 10^{-29} Joule

$\Delta\phi = Ut_p/\hbar = \pi$

- Hannover expt







Dark soliton propagates with speed less than speed of sound, 3.7 mm/s here

Would be at rest for perfect π phase shift

Dark soliton wavefunction

$$\Psi_0(x) = \sqrt{n_0}\left[i\frac{v_k}{v_s} + \sqrt{1-\frac{v_k^2}{v_s^2}}\tanh\left(\frac{x-x_k}{l_0}\sqrt{1-\frac{v_k^2}{v_s^2}}\right)\right]$$

n_0: condensate density

v_k: soliton velocity

x_k: soliton position

v_s: speed of sound $\quad v_s = \sqrt{4\pi a n_0}\hbar / m$

l_0: healing length $\quad l_0 = (4\pi a n_0)^{-1/2}$

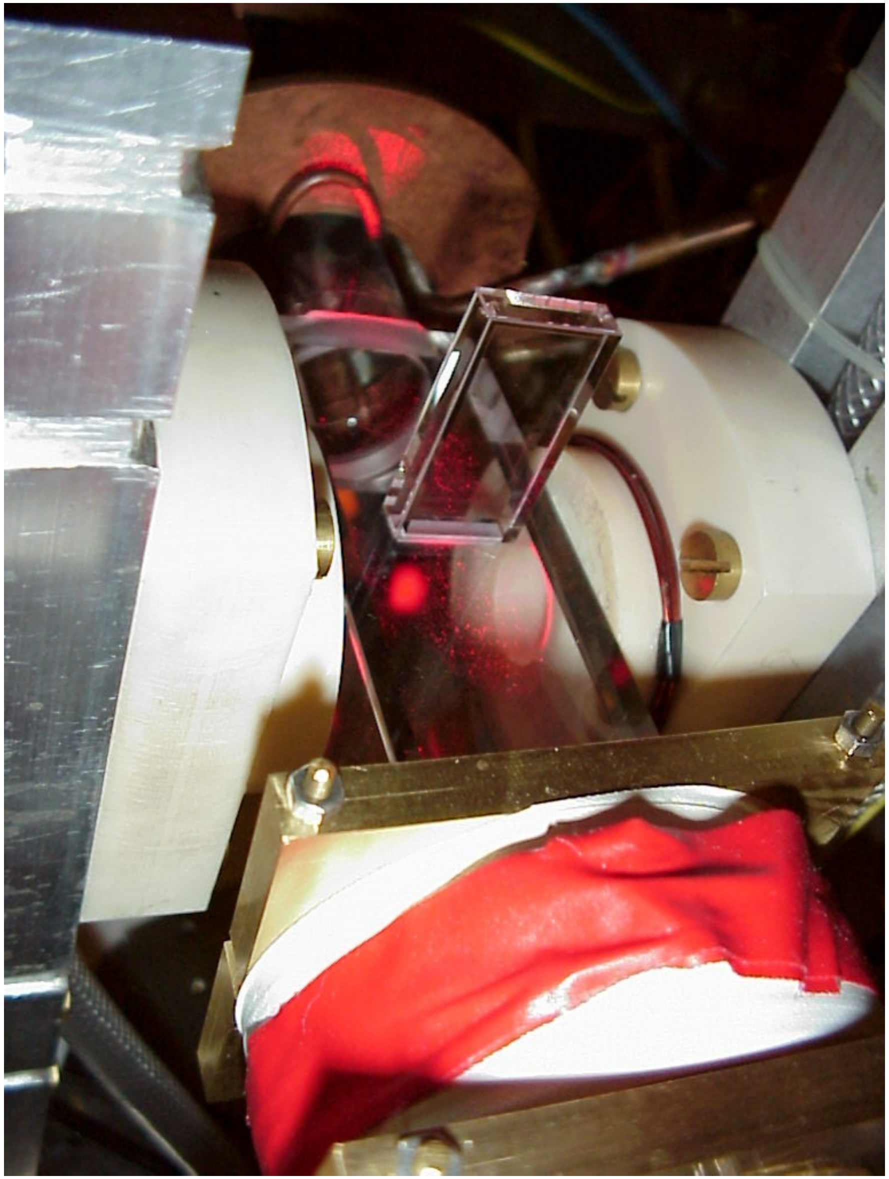
Soliton disappears at condensate edge

Thermally or dynamically unstable

$a<0$: Bright Matter Wave Solitons in Lithium 7

ENS, L. Khaykovich et al., Science **296**, (2002)
Rice University, K. Strecker et al., Nature **417**, 150 (2002)

Lithium Magneto-optical Trap

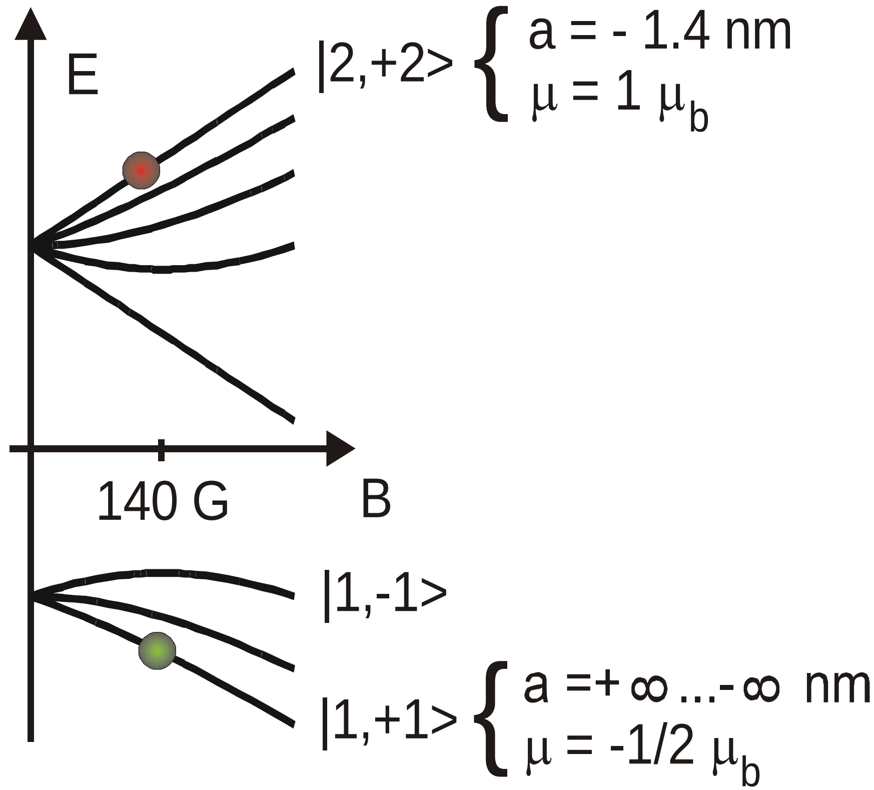


	Number	T [μK]
^{7}Li	10^{10}	1000
^{6}Li	10^{7}	700

Schreck et al., PR A 61, 011403R

Li 7 Bosons

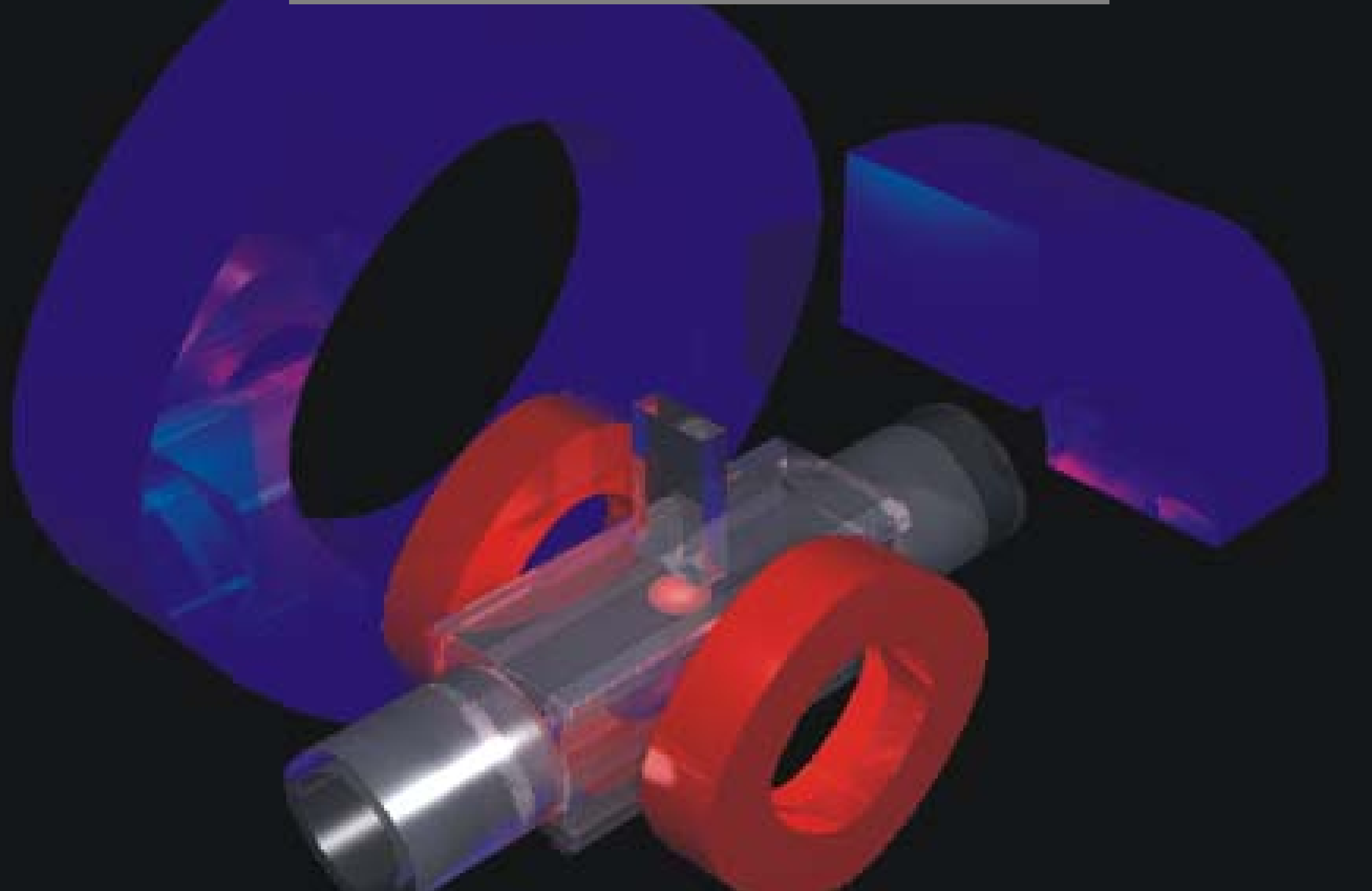
^7Li



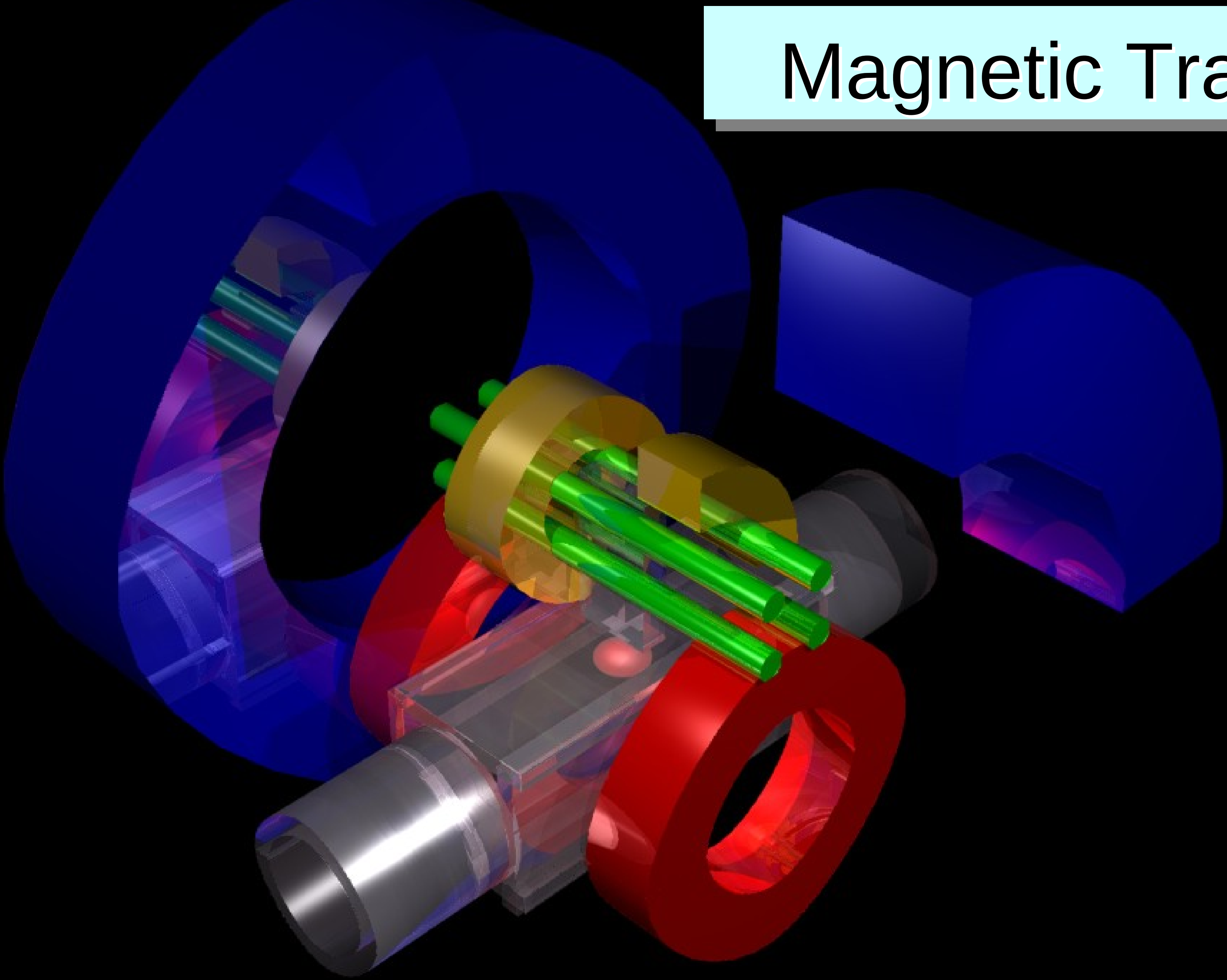
Feshbach resonance

Change of microscopic collision properties by an external magnetic field.

The Magnetic elevator



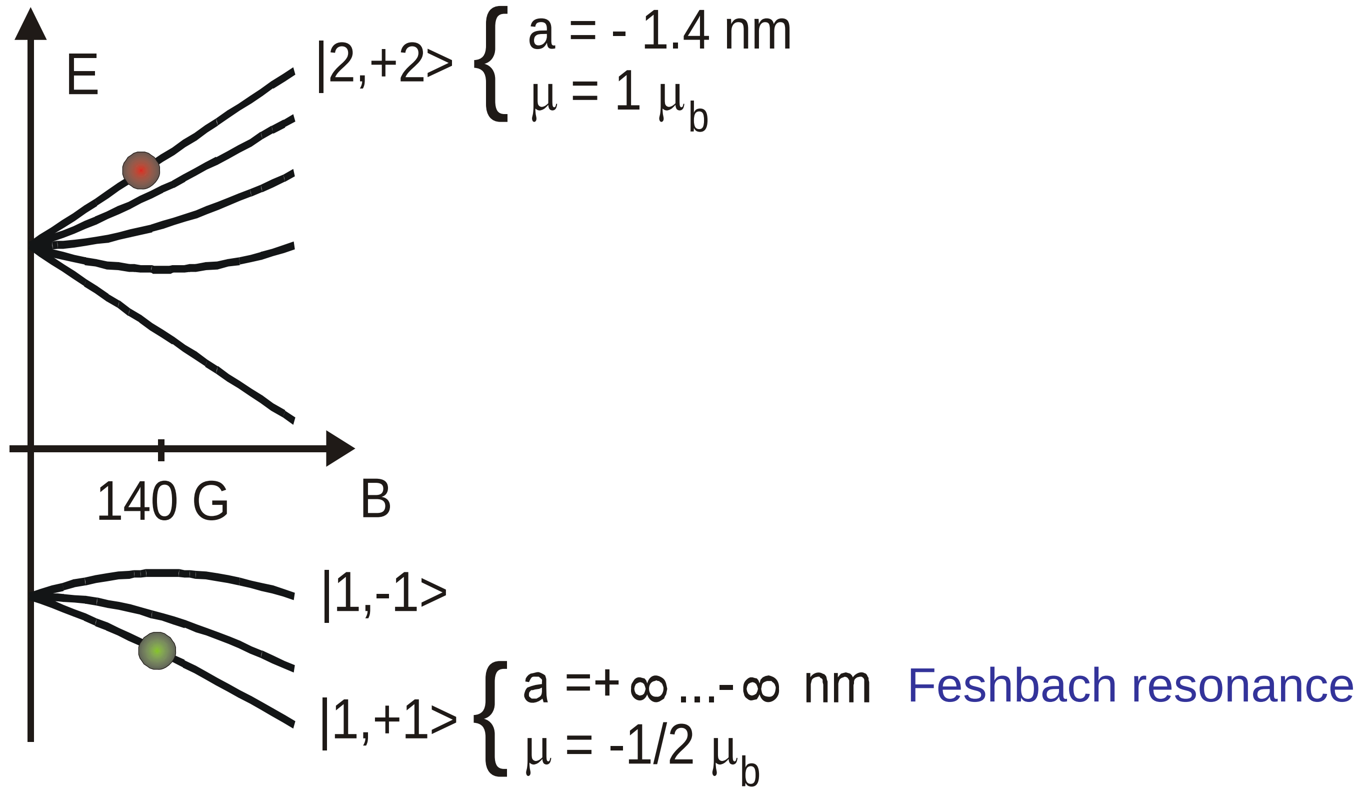
Magnetic Trap



- $\omega_{ax} = 2\ \pi\ 130$ Hz , $\omega_{rad} = 2\ \pi\ 8000$ Hz

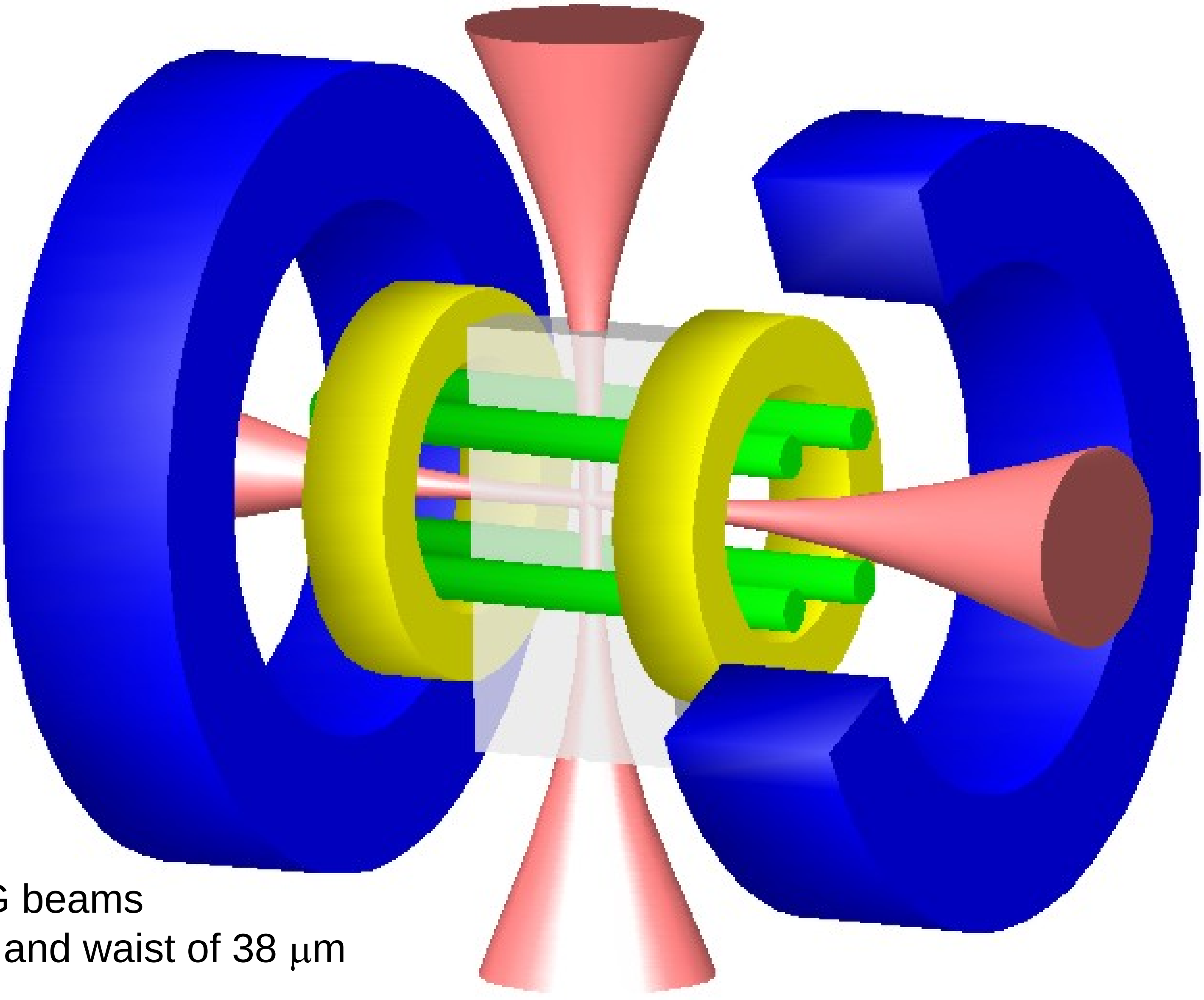
Li 7 Bosons

^7Li



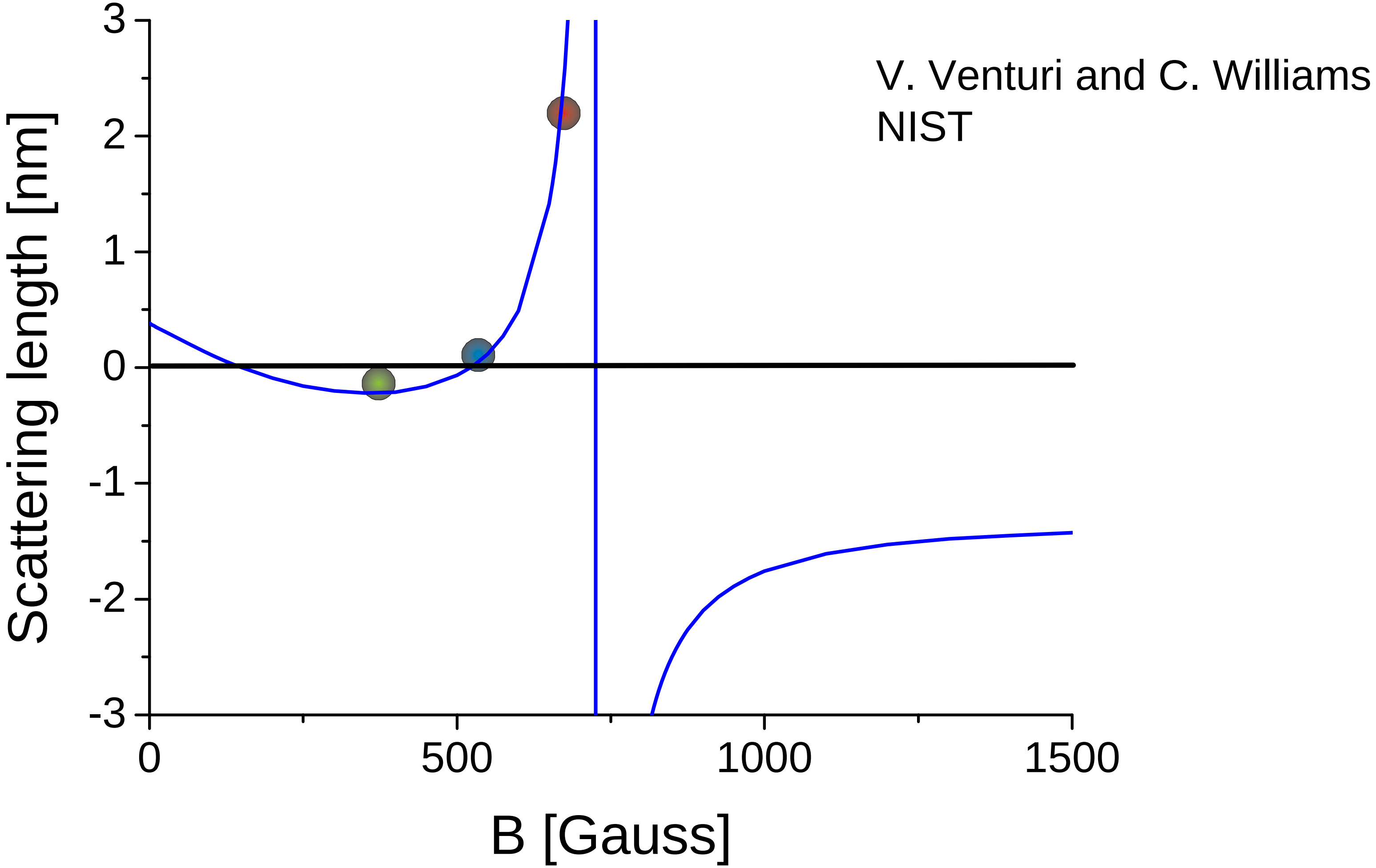
Change of microscopic collision properties by an external magnetic field.

The crossed dipole trap



Two YAG beams
with 5W and waist of 38 μm

Feshbach resonance in 7 Li |F = 1, m_F = 1>



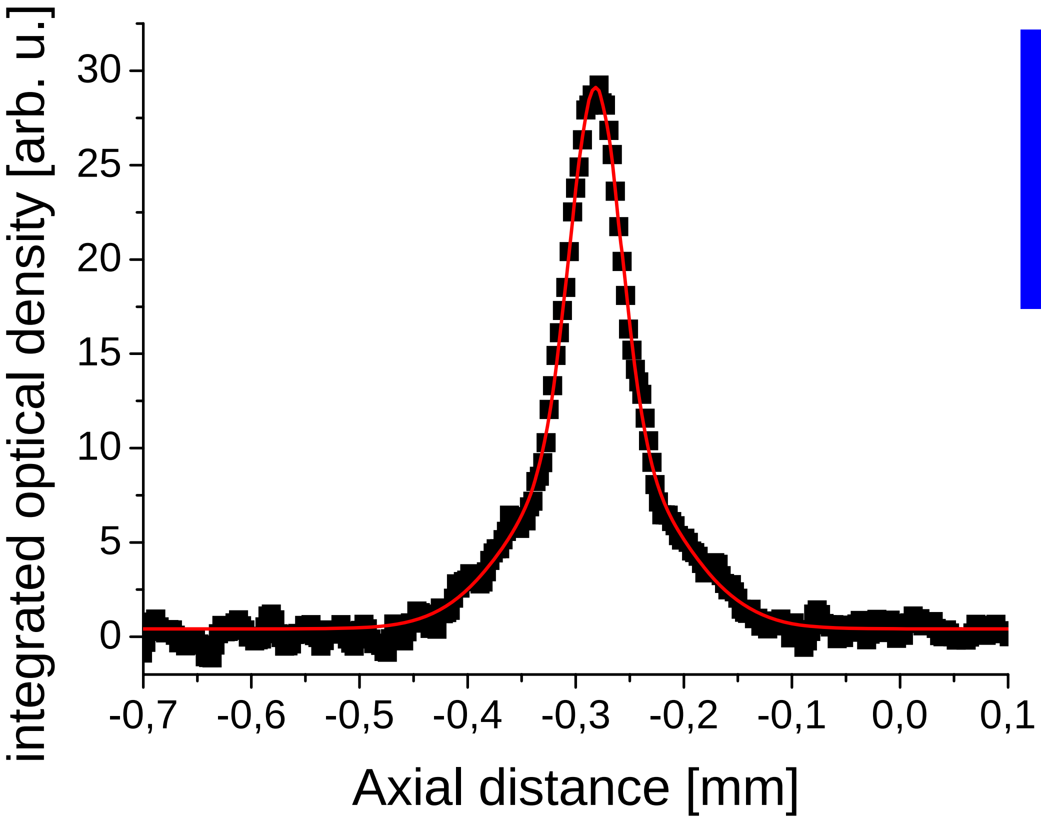
Evaporation

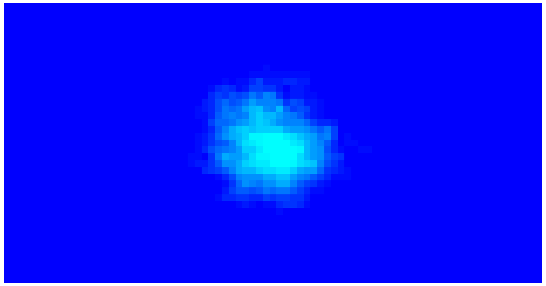
Gas without interactions: Ideal gas

Scattering length a < 0: attractive interactions

^{7}Li Condensate in optical trap with adjustable a

- Evaporation to 10 μK in magnetic trap
- Transfer atoms into dipole trap
- Transfer from |2,2> to |1,1>
- Evaporation to BEC with a = + 2.5 nm
- Reduction of trap depth by x20 in 250 ms !!!

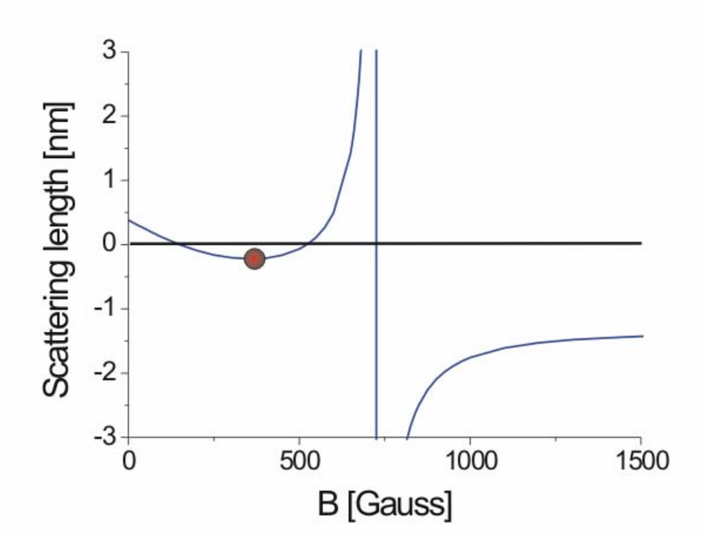




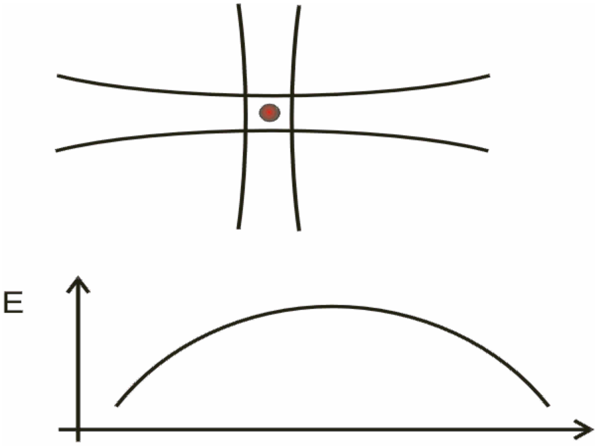
Nc= 2 10^4
At T=Tc / 2

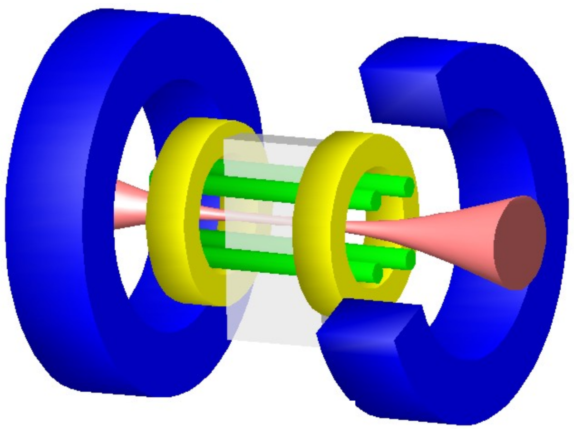
Soliton production

- 1D gas with dispersion counterbalanced by non linear interaction
- Change scattering length

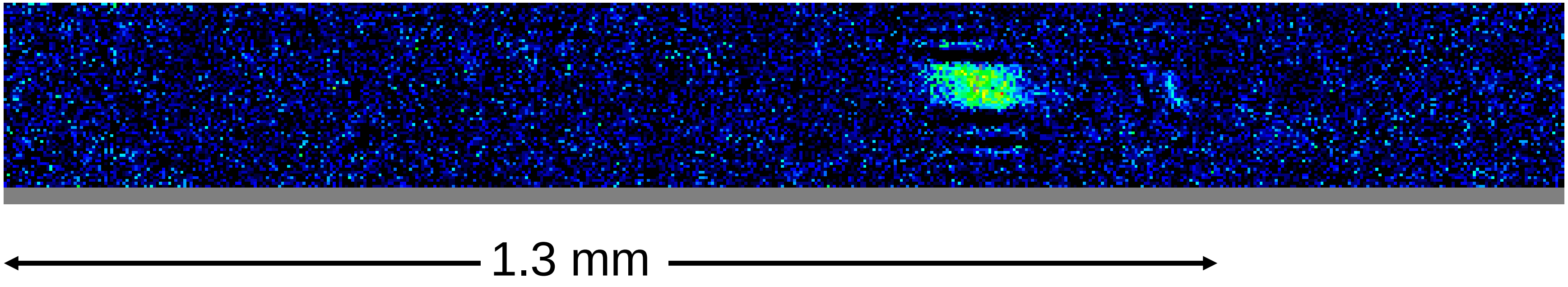


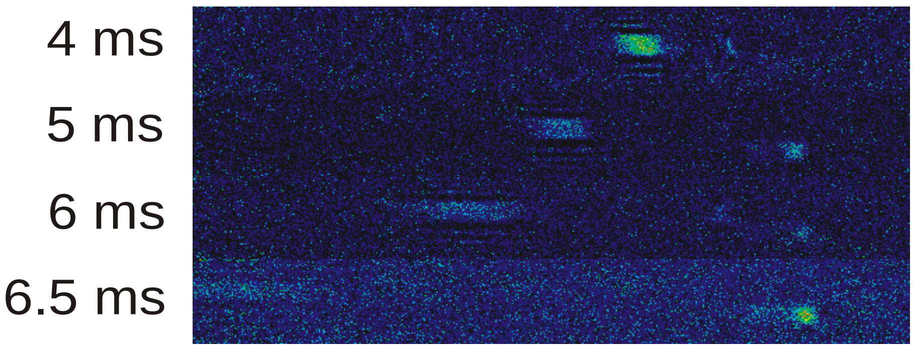
- Cut axial confinement and observe expansion in expulsive axial potential

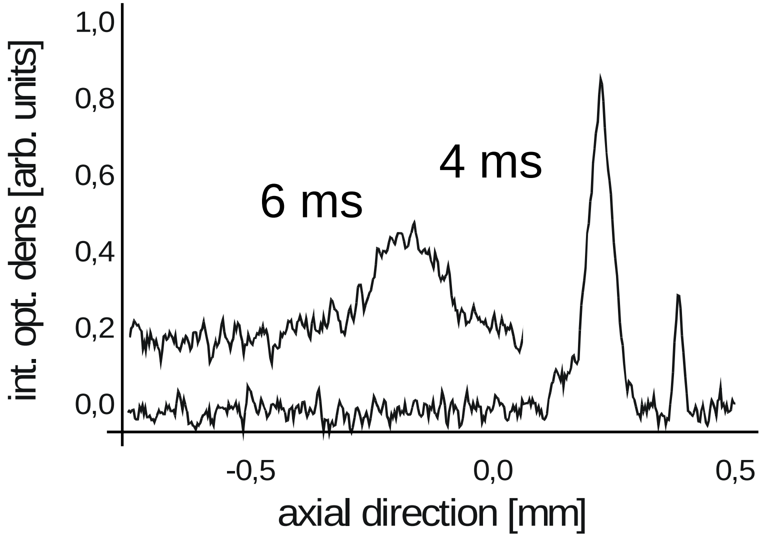




Ideal gas in 1D waveguide: a~0

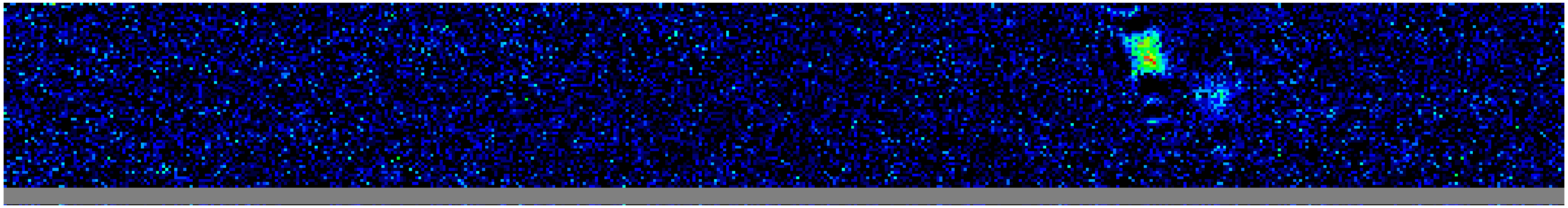




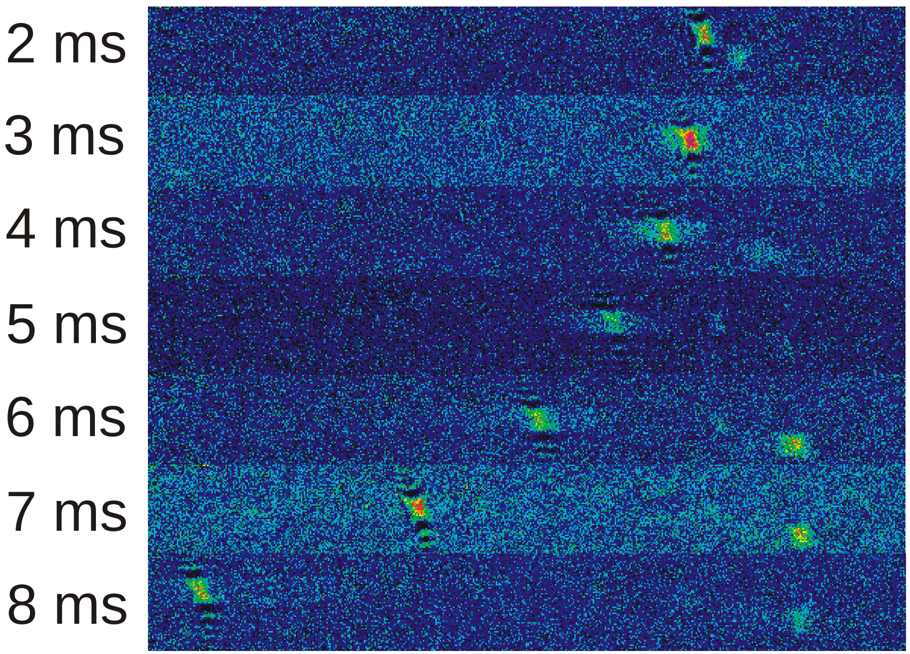


Dispersion of non interacting matter waves

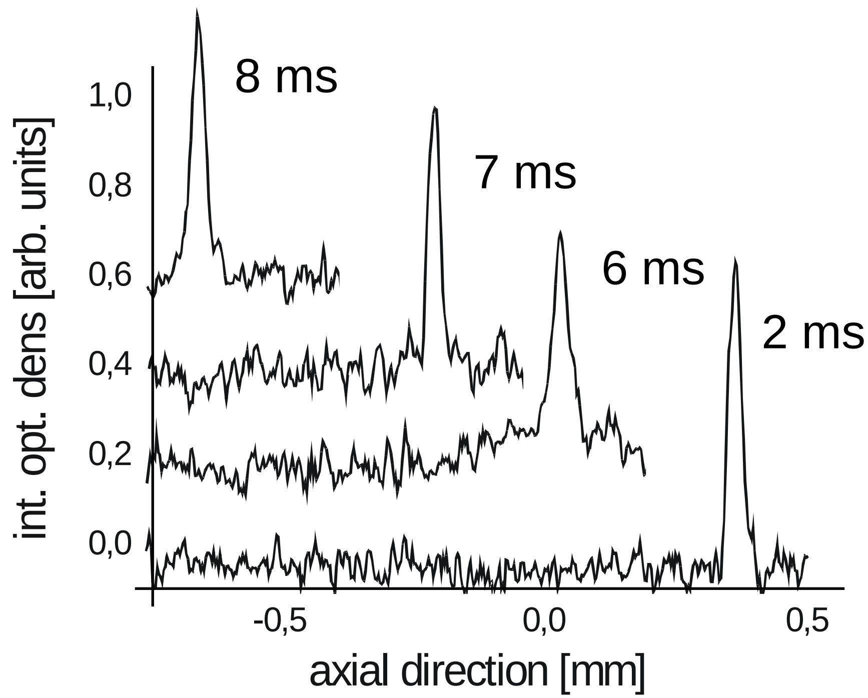
Soliton : a < 0



1.3 mm

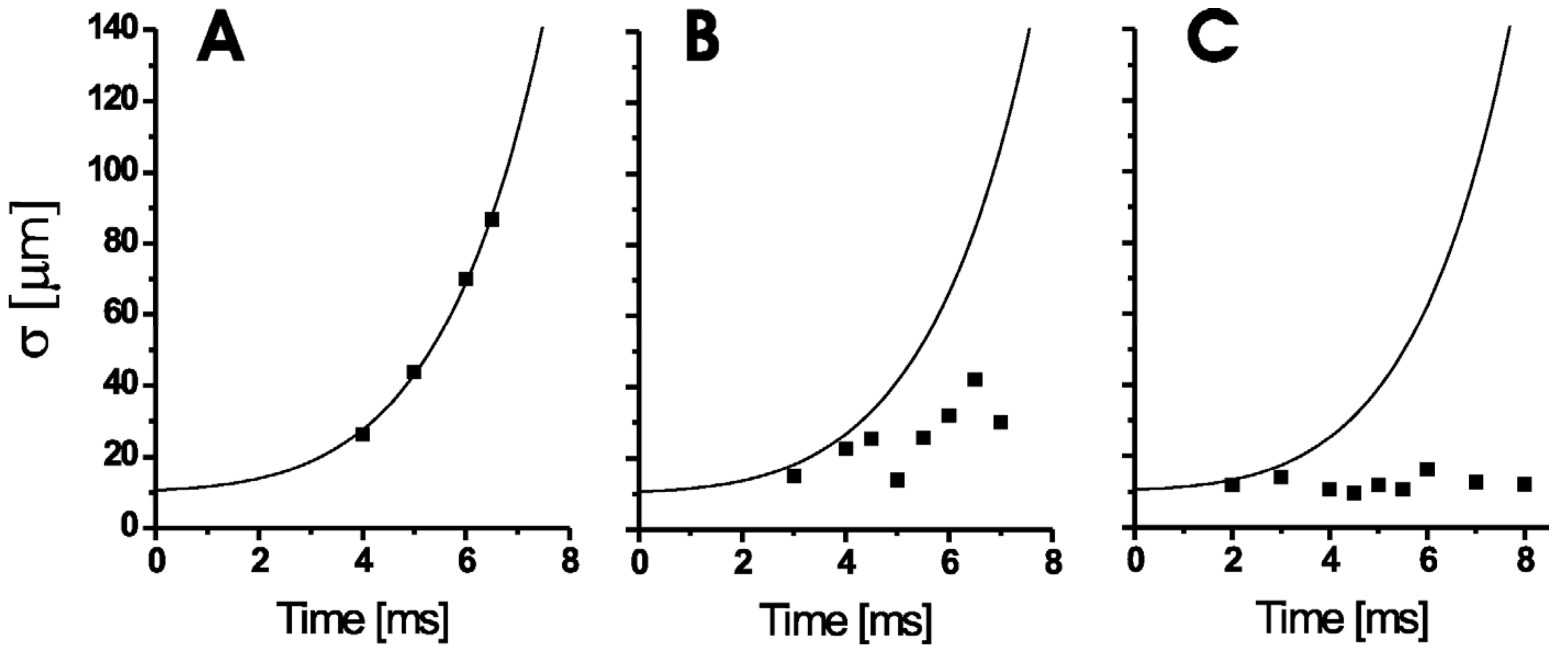


Propagation without spread of
N= 6(2) 10^3 atoms



L. Khaykovich et al., Science 296, 1290, 2002

Ideal gas versus Soliton



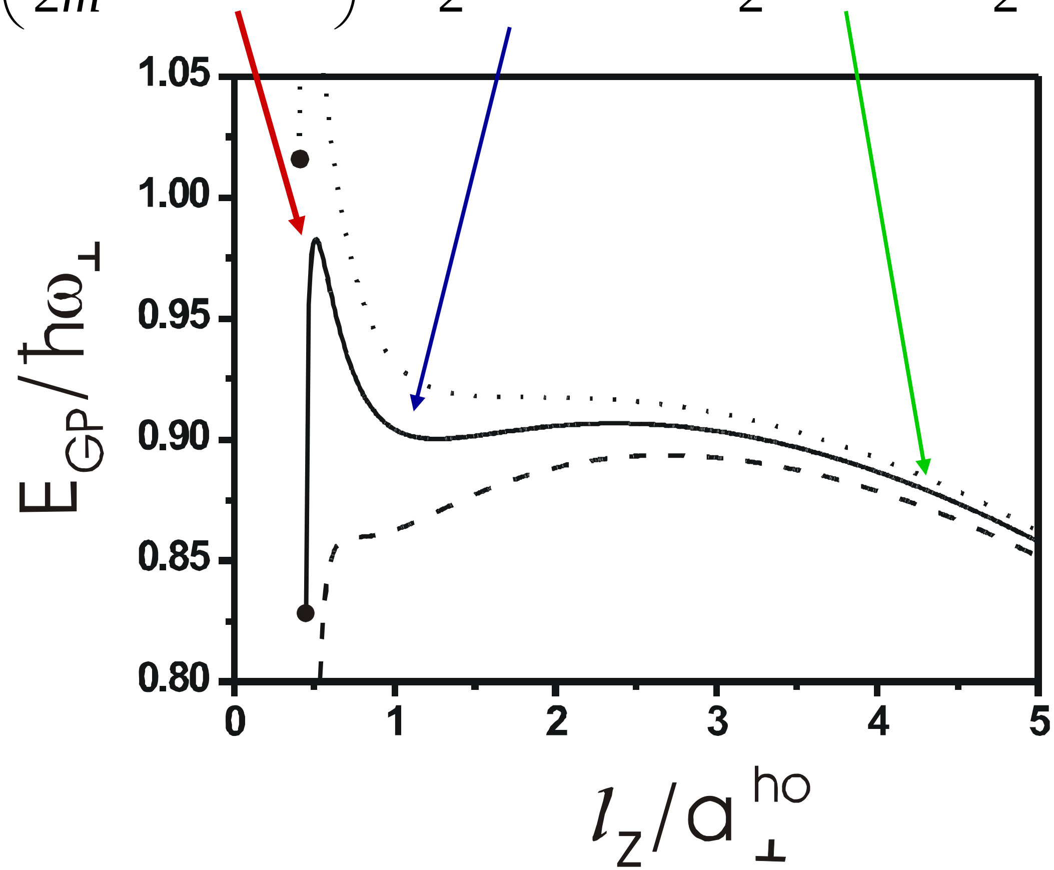
a ~ 0

a= - 0.11 nm

a = - 0.21 nm

Stability diagram of the soliton: solution of 3D GP equation

$$E_{GP}[\psi] = \int d^3\vec{r}\left(\frac{\hbar^2}{2m}|\nabla\psi(\vec{r})|^2\right) + \frac{gN}{2}|\psi(\vec{r})|^4 + [\frac{1}{2}m\omega_z^{\ 2}z^2 + \frac{1}{2}m\omega_\rho^{\ 2}(x^2+y^2)]|\psi(\vec{r})|^2$$



For a= -0.21 nm

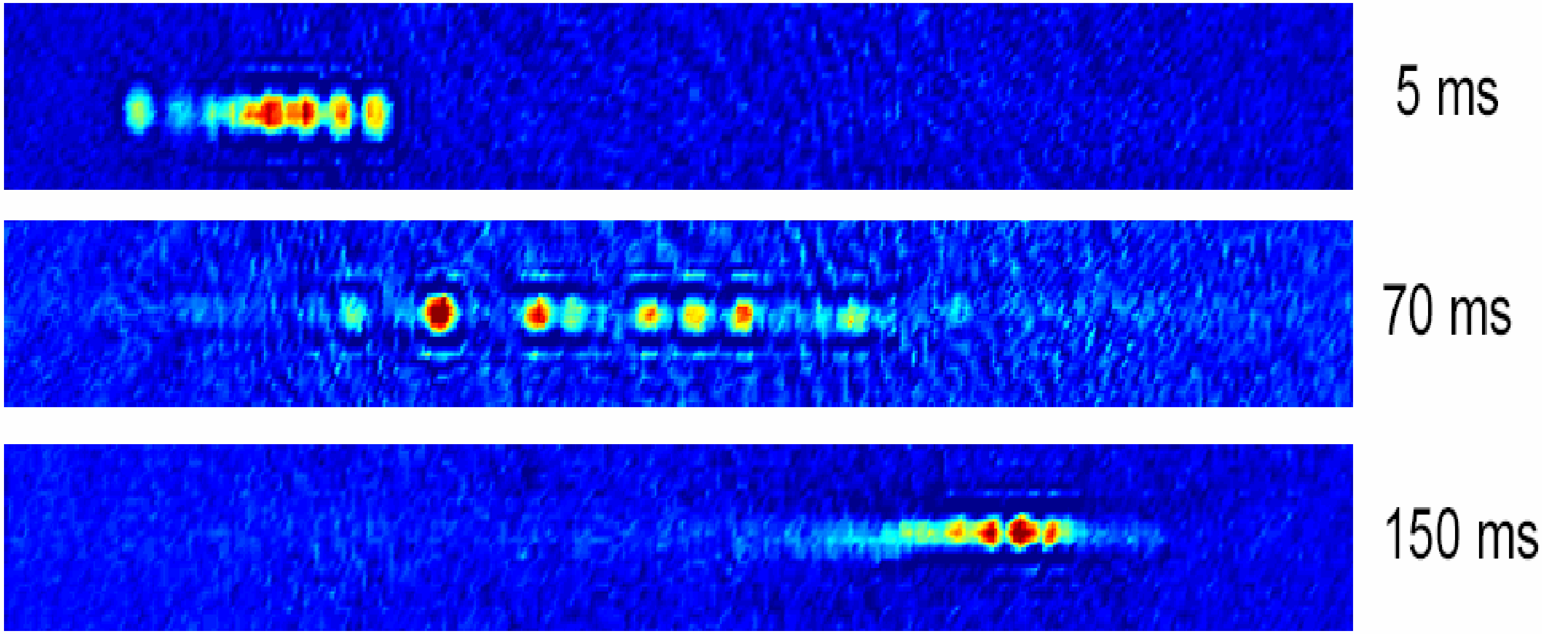
$\omega_z/\omega_\perp = i\times 70/710$

Expected: $4.2\times10^3 \le N_{at} \le 5.2\times10^3$ $\quad l_z = 1.7\,\mu m$

Measured: $N_{at} = 6(2)\times10^3$

Soliton Trains

Rice University, K. Strecker et al., Nature **417**, 150 (2002)



Contrarily to the ENS experiment, solitons are always formed in numbers greater than 4 !

They oscillates up to 3 seconds in the optical trap; some disappear

Evidence for soliton repulsion between neighboring solitons

Solitons formed with same relative phase attract each other

Solitons formed with π relative phase repel

Soliton train formation: a possible scenario

L. Carr & J. Brand
PRL 92, 2004

Formation process: modulation instability:

Attraction creates density instability over spatial scale of order of the healing length,

$$\xi = (8\pi n|a|)^{-1/2}$$

creating a sequence of local collapses, in which the wavefunction amplitude and phase re-arrange in a more stable configuration of solitons with alternating phases.

Dynamics still largely unexplored !

Return on Collapse of 3D BEC

Collapse of 3D condensates

Are solitons really in 1D regime ?
Role of trap anisotropy
Cornish, Thomson, Wieman, PRL 96 (2006)

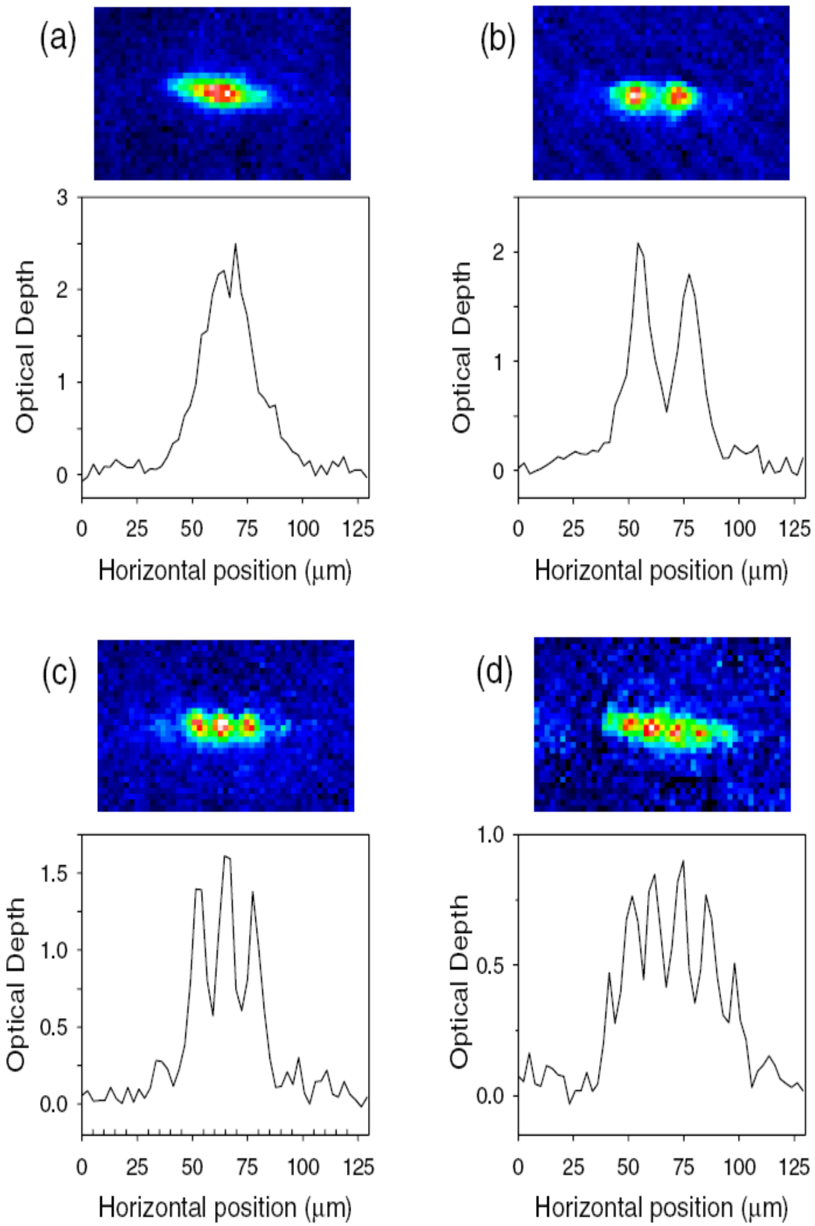
JILA expt's on 85Rb $\nu_r/\nu_z \approx 2.5.$

Prepare Rb 85 condensate with N_0=8000 atoms on a>0 side
Then switch to a<0,
N_0 is clearly greater than max number of atoms in soliton

$$N_0 < N_{\mathrm{critical}} = k\frac{a_{\mathrm{ho}}}{|a|},$$

Wait adjustable time up to several 100 ms
Take a time of flight image. What do you see ?

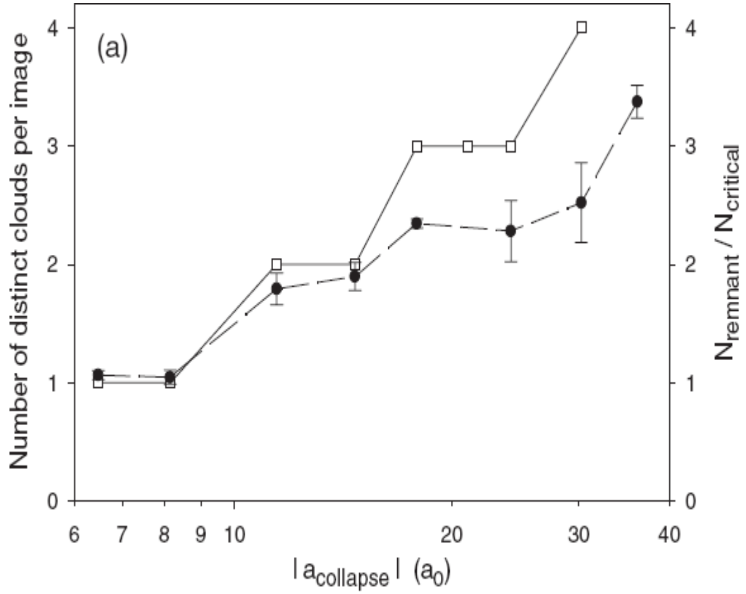
Solitons formed during collapse

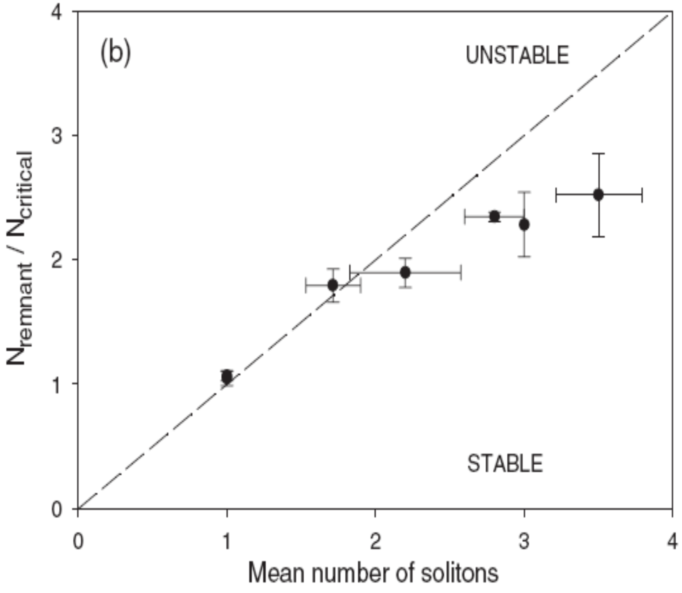


Fraction of atoms surviving the collapse ranges from 60 % for a ~ -5a_0 to 30% For a=-50a_0. Other atoms disappear from the trap (or form a thermal cloud).

Controllable production of the number of solitons

Soliton number vs scattering length

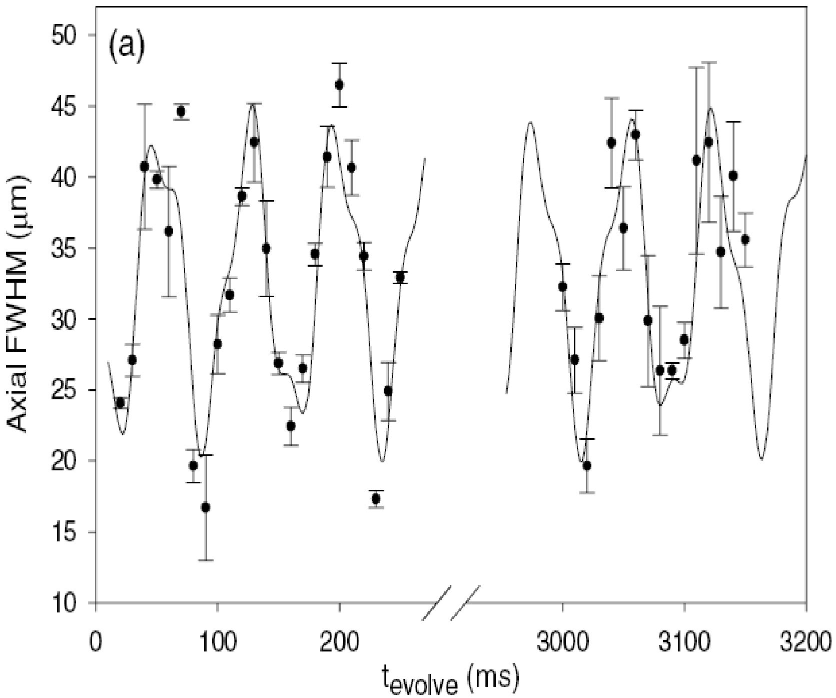


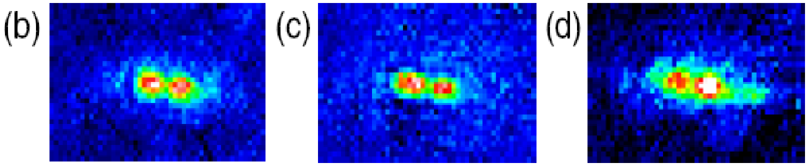


$$N_{\mathrm{critical}} = k\frac{a_{\mathrm{ho}}}{|a|}$$

k= 0.46

Two Solitons oscillating in magnetic trap





Neighboring solitons are formed with relative phase ensuring repulsion

$\pi/2 < \phi < 3\pi/2)$

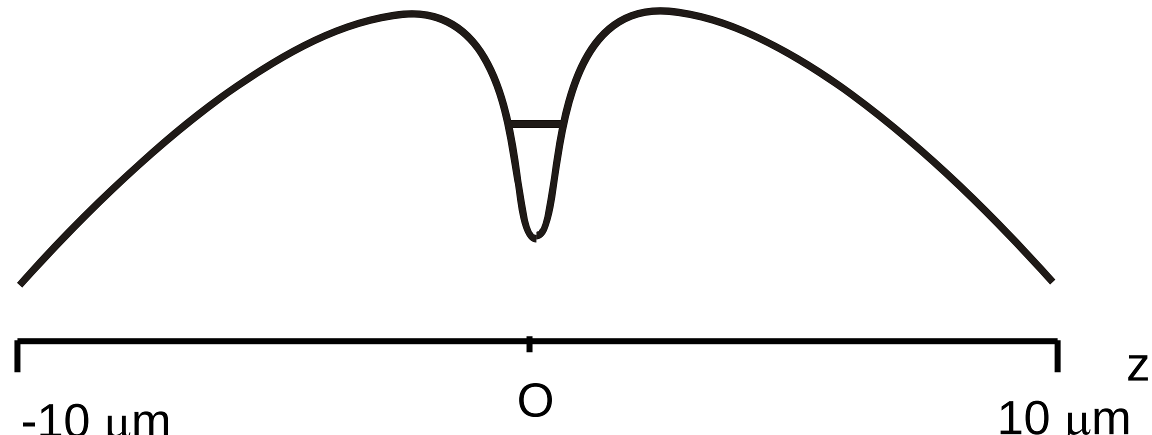
Two Solitons never fully overlap
Never reach critical collapse
And remain stable

Confirmed by numerical simulation of 3D GPE equation

Two solitons oscillating for 3 seconds
And repelling each other

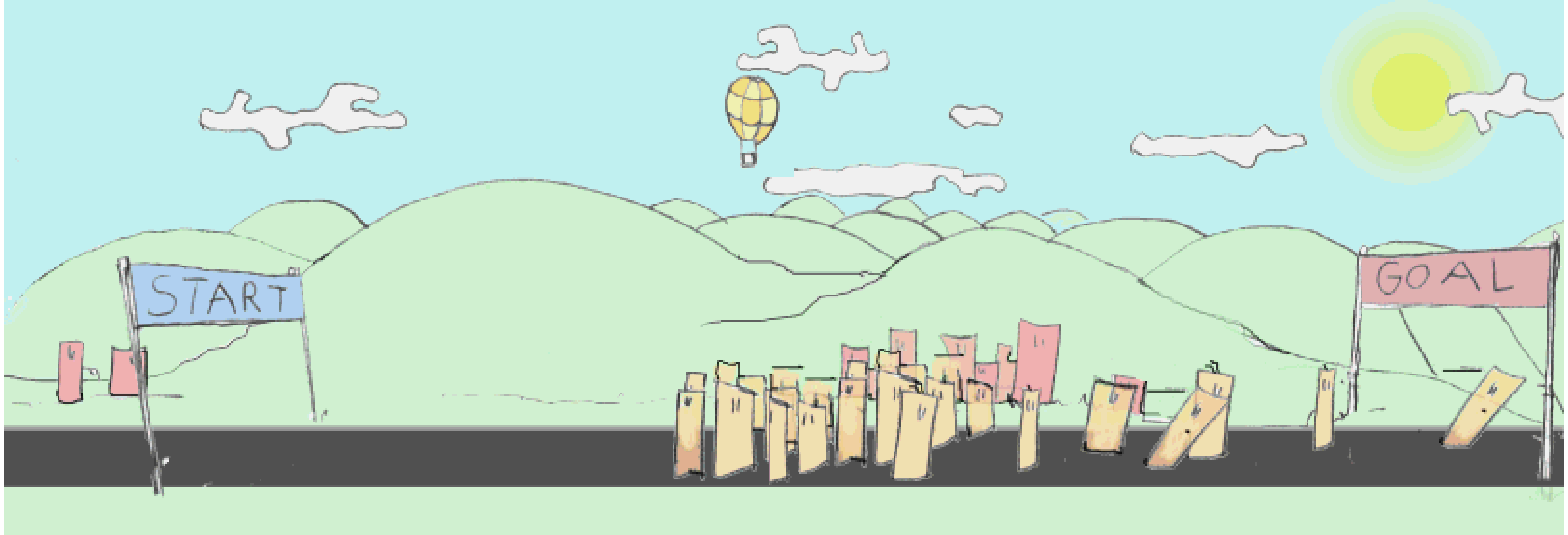
Perspectives on Solitons

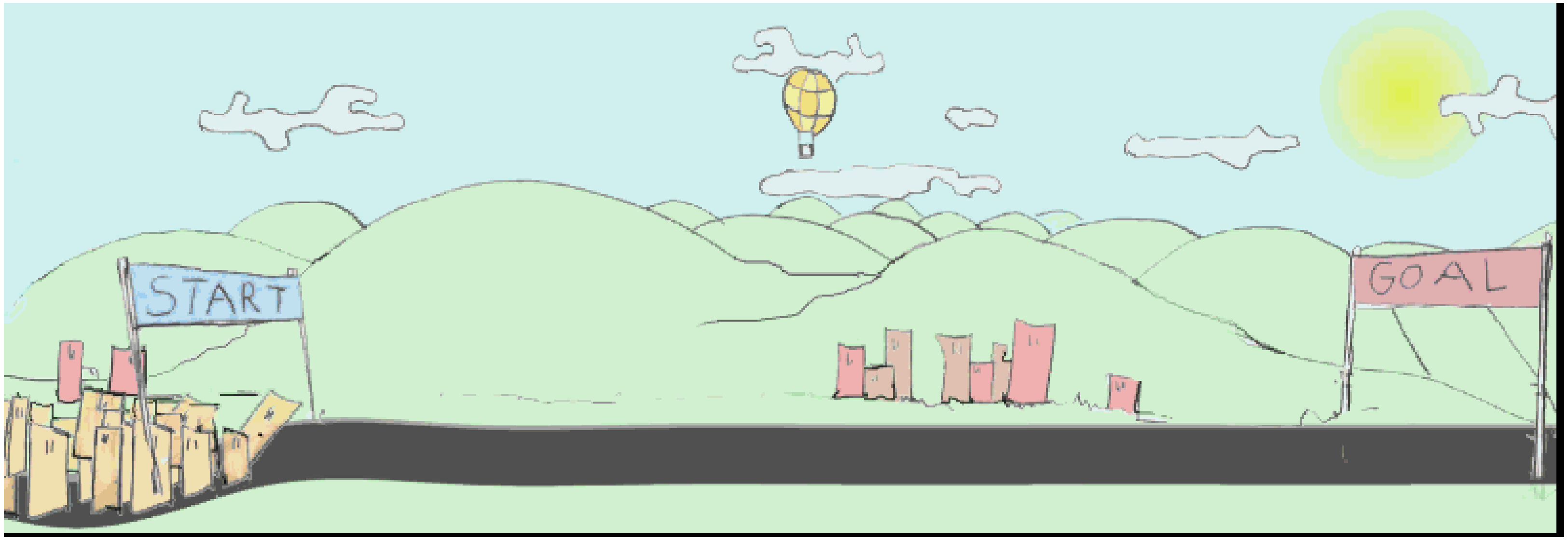
- Vast litterature on solitons, vortex rings, vortex pairs, in 2D and 3D structures, JILA, Harvard experiments
- Nice review by L. Carr and J. Brand: archiv 0705/1139
- Dynamics of the soliton formation
- Soliton with small (N=2, 3, …) atom numbers: pairs of correlated atoms; EPR tests with massive particles
- Soliton atom laser and interferometry with solitons
- Production of higher order solitons:
- Shifting the scattering length by a factor 4 would produce a soliton of order 2: time dependent oscillation of soliton amplitude and phase
- A soliton decay process: quantum evaporation through a tunnel barrier which depends on N: non linear quantum tunneling

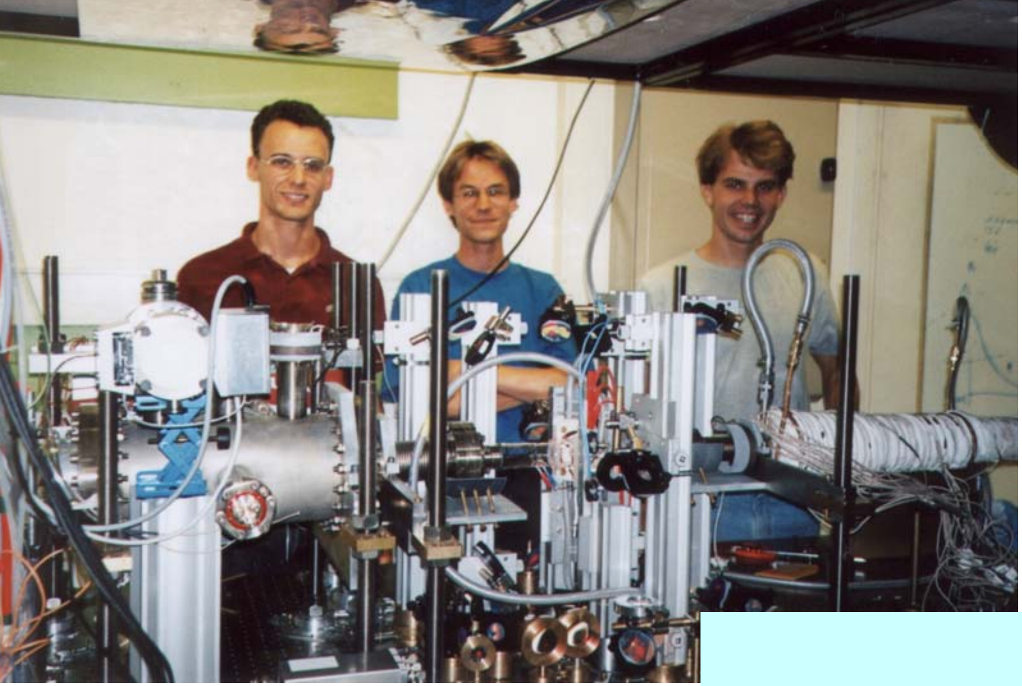


L. Carr
Y. Castin

Soliton of Marathon runners in Greece







Thanks !