Sharp improved weighted Hardy-Kato inequalities for anisotropic elliptic problems

Konstantinos Tzirakis[1*#]

[1]Institute of Applied and Computational Mathematics, Foundation for Research and Technology - Hellas, Heraklion Crete, Greece

Presenting author: Konstantinos Tzirakis, email: kostas.tzirakis@gmail.com

* Corresponding author: Konstantinos Tzirakis, email: kostas.tzirakis@gmail.com

ABSTRACT

The poster presents recent developments in anisotropic weighted Hardy and trace Hardy inequalities – with the anisotropy being represented by a generic Finsler metric – which are associated to the so-called Finsler or anisotropic Laplacian. The Finsler Laplacian is one of the most natural and foremost operators in the theory of anisotropic and non-homogeneous media, and in Finsler or Minkowski geometry. Anisotropic variational problems arose in crystallography, as minimization of anisotropic surface tensions for determining of equilibrium shapes of crystals; their study, was initiated in the historical work of George Yuri Viktorovich Wulff using a geometrical construction. The relative mathematical theory was later developed, including analytic and geometric aspects, and attracts increasingly many attentions in the last decade, due to its still further applications to other branches of physics, in biology and other fields.

Following a unifying approach, we establish first a sharp interpolation between weighted Hardy and trace Hardy inequalities, extending the corresponding non-weighted version, being established recently by a different approach. Then, passing to bounded domains, we obtain successive sharp improvements by adding correction terms involving sharp weights and optimal constants, resulting in an infinite series-type improvement. The generalization of our results to cones is also included.