Topics in Network Economics

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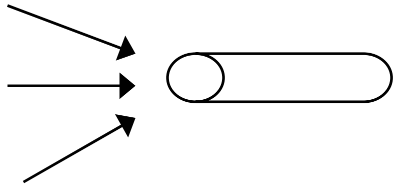
Economics = incentives

- The taxi tariff $w = a + bT + cX$
- The “all-you can eat” restaurants: flat vs usage-based
- The Internet café tariff: dynamic pricing
- Pricing a single link



Questions

- Over-dimensioning of networks?
- Will congestion exist in the future?
- How demand will grow?
- What will be the future applications?
- Will the “real” Internet ever exist?
- Telecommunications network just like the electrical?
- Power of position in the value chain?

Course outline

- Consumer and producer model: utility and demand function, cost and production function, social welfare and marginal cost pricing
- Application in networks: charging as a control mechanism, examples
- Externalities, congestion pricing, p2p
- Information
- Cost recovery

Basic economic concepts

The context

- Communication services are **economic goods**
- **Demand factors:** amounts of services purchased by users
  - utility of using a service, demand elasticity
- **Supply factors:** amounts of services produced
  - technology of network elements, service control architecture, cost of production
- **Market model:** models interaction and competition
- **Prices:** control mechanism
  - control demand and production, deter new entry
  - provide income to cover costs
  - structure and value depends on underlying model

Economic models and tariffs

- Prices result from the solution of economic models
- Three major contexts for deriving optimal prices
  - **surplus maximization**: standard market models with ***actual*** competition: monopoly, oligopoly, perfect competition
  - **stability under competition and fairness**: sustainability against ***potential*** entry, recovering costs, fairness w.r.t. cost causation, no subsidization
  - **asymmetric information models**: principal-agent models, hidden action and hidden information

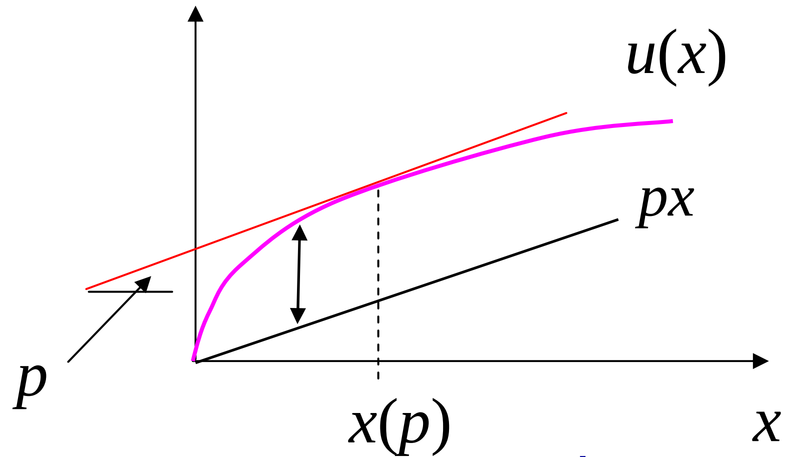
The consumer

The consumer’s problem

- **Consumers:**
  - utility function $u(x)$ increasing, concave
  - **consumer surplus** (net benefit): $u(x) -$ charge for x
  - solve optimisation problem (linear prices):

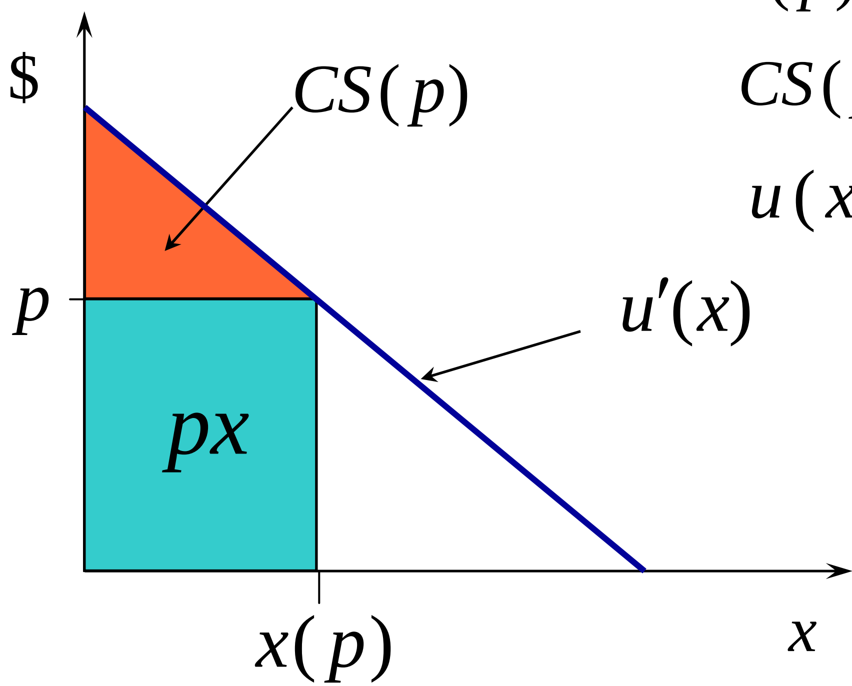
$$x(p) = \arg\max[u(x) - px]$$

  - at optimum $p = u'(x)$



The demand curve

The demand curve: $D(p)$



$x(p) =$ quantity demanded at price p

$CS(p) =$ consumer surplus at price p

$u(x) = CS(p) + px$

= value of consuming x

$$x(p) := \arg\max\{u(x) - px\}$$

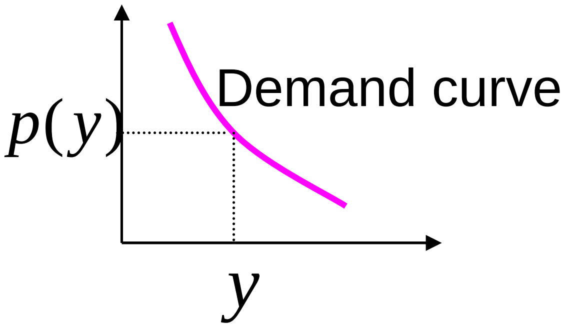
The producer

The producer's problem

- **Producer:** profit function (**producer surplus**):

$$\pi(y) = yp(y) - c(y),\ y \in Y$$

Monopoly: $\max_{y \in Y} [\, p(y)\, y - c(y)]$



Perfect competition: $\max_{y \in Y} [\, py - c(y)],\ \text{for given}\ \ p$

Oligopoly: $\max_{y \in Y} [\, p(y + \boxed{z})\, y - c(y)]$

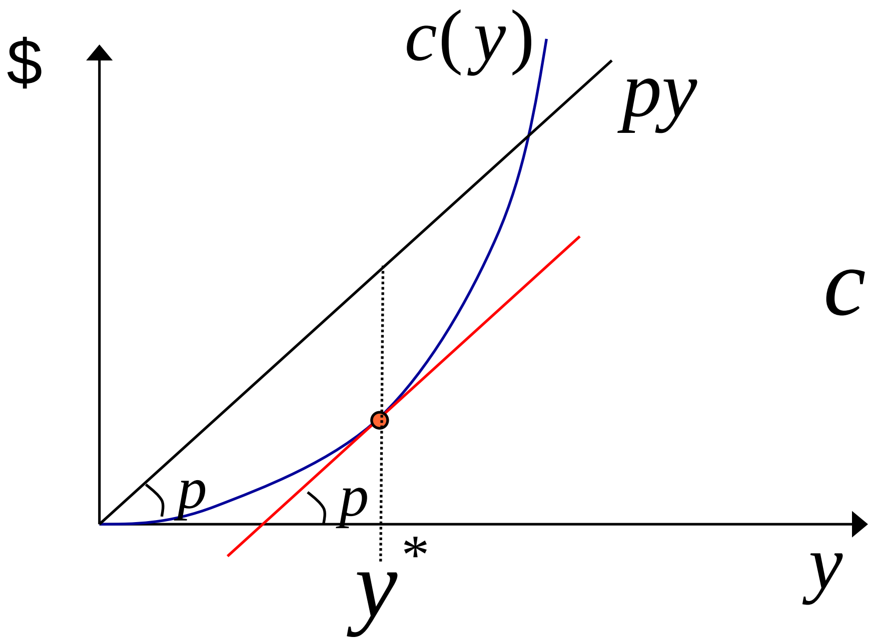
Regulation: fixed p, produce $y = y(\, p\,)$

The producer in a competitive market

Competitive market with price $\overline{p}$:

$$D(p) = \begin{cases} 0 \text{ if } \; p > \overline{p} \\ \text{any amount produced if } \; p = \overline{p} \\ \infty \;\text{ if } \; p < \overline{p} \end{cases}$$

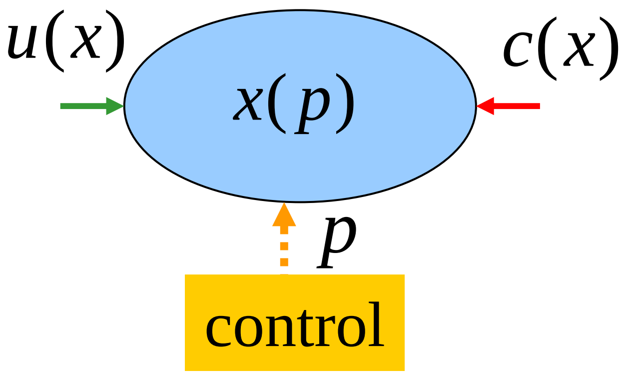
Producer solves: $\max_{y} \; py - c(y) \;$ for $p = \overline{p}$



$$c'(y^*) = p$$

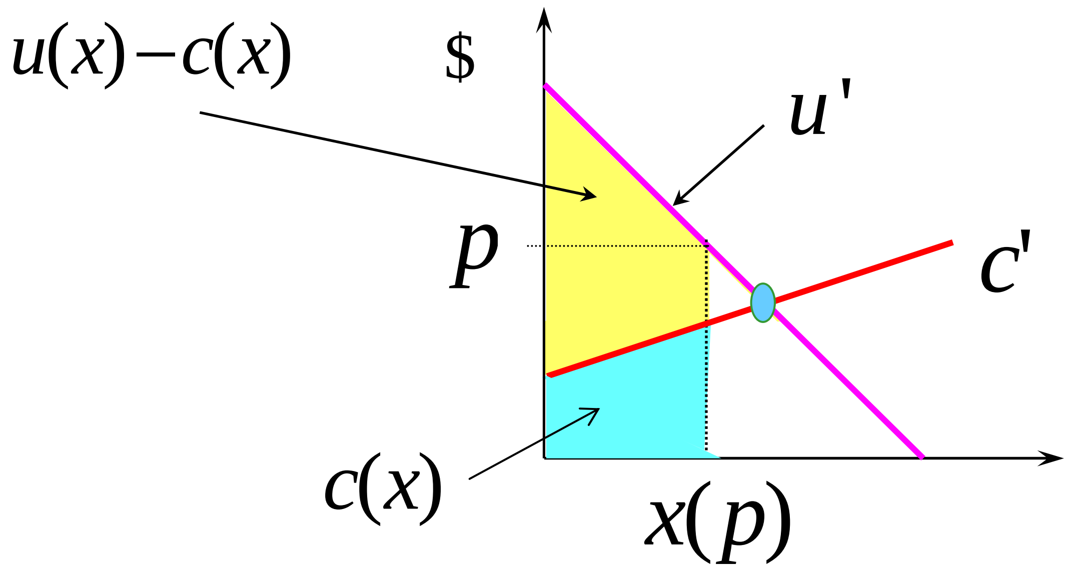
The social planner

The social planner's problem



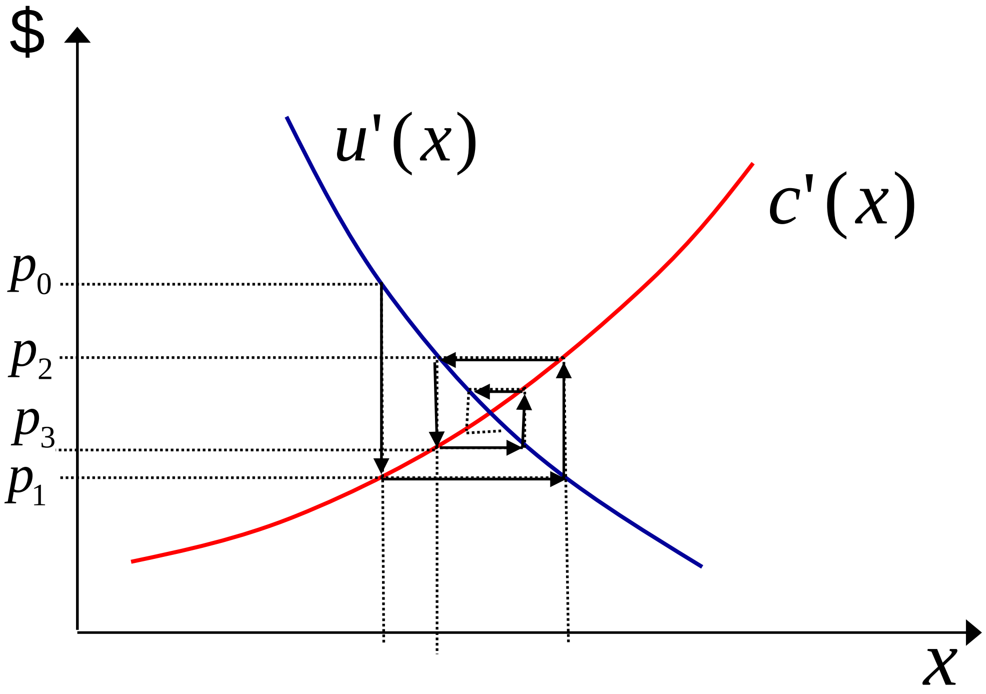
$$\max_x \; u(x) - c(x) \Leftrightarrow$$

$$\frac{\partial u(x^*)}{\partial x_i} = \frac{\partial c(x^*)}{\partial x_i}$$ **<=> MU = MC**



Setting prices equal to marginal cost

- The social planner sets prices equal to marginal cost at the level of production that satisfies demand
- Prices (may) converge to SW optimum

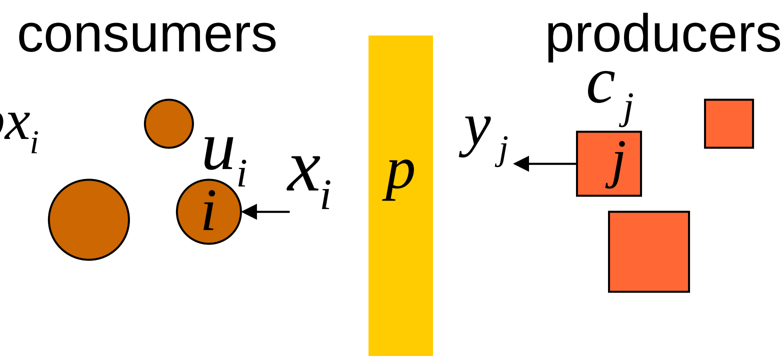


Market mechanisms and competitive equilibria

Competitive equilibrium

- Every participant in the market is small, can not affect prices
- Equilibrium: stable point where production = demand, price *p*

$$\max_{x_i} \; u_i(x_i) - p x_i$$



$$\max_{y_j} \; p y_j - c_j(y_j)$$

Market clearance: $\sum_i x_i(p) = \sum_j y_j(p)$

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial c_j}{\partial y_j} = p \Leftrightarrow \max_{\{x_i, y_j\}} \sum_i u_i(x_i) - \sum_j c_j(y_j)$$

=> **Social welfare optimum!**

$$s.t. \quad \sum_i x_i \le \sum_j y_j$$

Capacity constraints

- Total amount of resource available = C
- Maximization problem:

$$\max_{\{x_i\}} \sum_i u_i(x_i) \quad s.t. \quad \sum_i x_i \le C \qquad (1)$$

- Mathematical solution: maximize the Lagrangian

$$\max_{\{x_i\}} L(\lambda, x_1, \ldots, x_n) = \sum_i u_i(x_i) - \lambda\left(\sum_i x_i - C\right)$$

The optimal point of (1) is characterized by $\lambda, \{x_i\}$ for which:

$$\sum_i x_i = C, \; \frac{\partial u_i}{\partial x_i} = \lambda$$

- Problem solution with market mechanism: use price $p = \lambda$
- Each user solves: $\frac{\partial u_i}{\partial x_i} = p$
- λ = **shadow cost** of capacity

Market mechanisms

1. Network sets price p^t, users post their demands $x_i^t(p^t)$

2. Network computes excess demand $z^t = \sum_i x_i^t - C$

3. Network updates price : $p^{t+1} = p^t + \alpha z^t,\ 0 < \alpha < 1$

Under general conditions, $p^t \to \lambda$

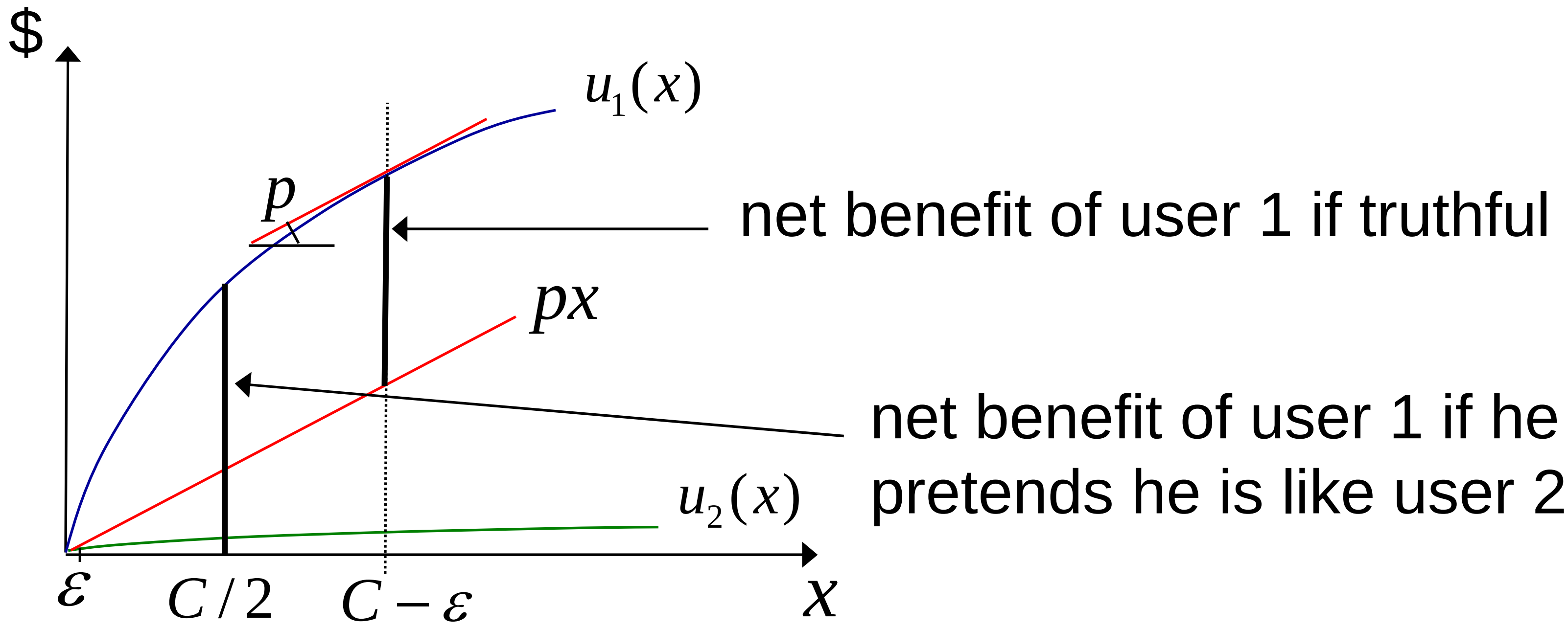
where λ is the Lagrange multiplier in (1)

Observe:

- The optimum of (1) is achieved by a decentralized mechanism
- The network does not need to know the utilities of the users

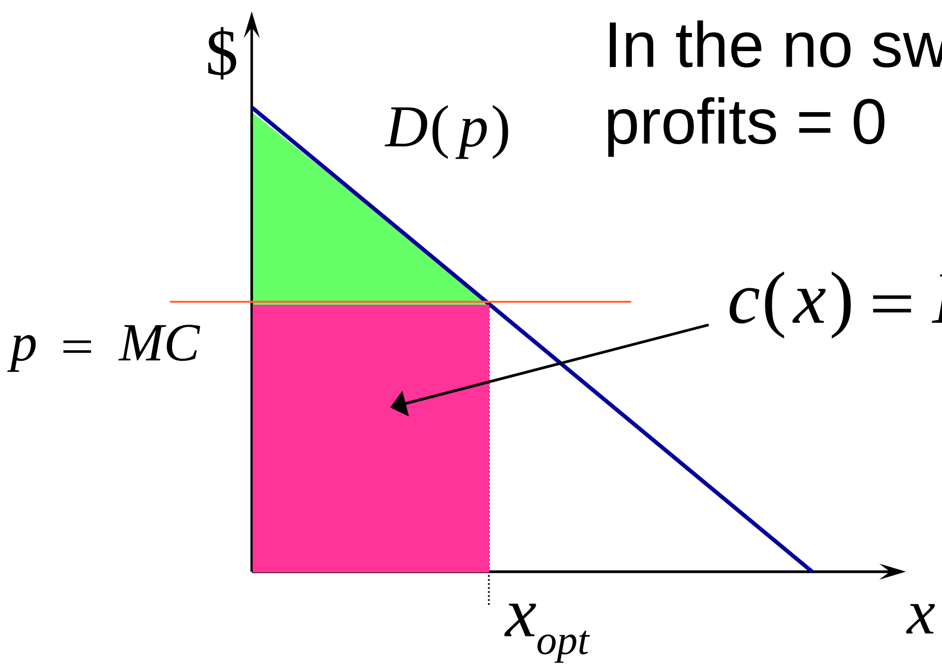
Strategy issues

- Why should users respond truthfully their $x_i(p)$?
- it may be profitable to cheat!
- In a case of 2 unequal users, the large user may pretend he is small



Lock-in generates profits

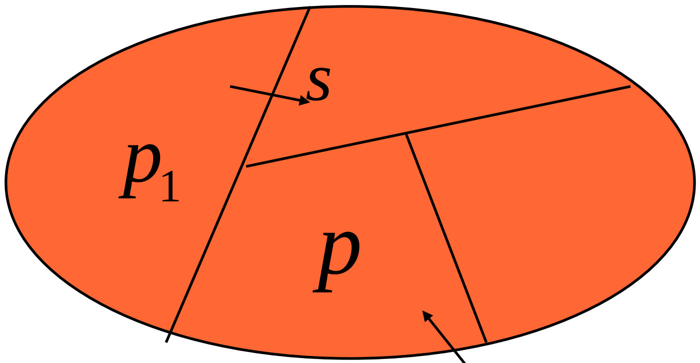
- Changing providers may involve switching costs
- These result in Lock-in: a provider may raise prices in equilibrium above marginal costs and still retain customers



In the no switching cost case profits = 0

$c(x) = MCx = \text{revenue}$

A model of switching cost



new entrant, offers discount d

$$p_1 + \frac{p_1}{r} = p + \frac{p}{r} + s - d, \quad \longleftarrow \text{ customer indifferent to switch}$$

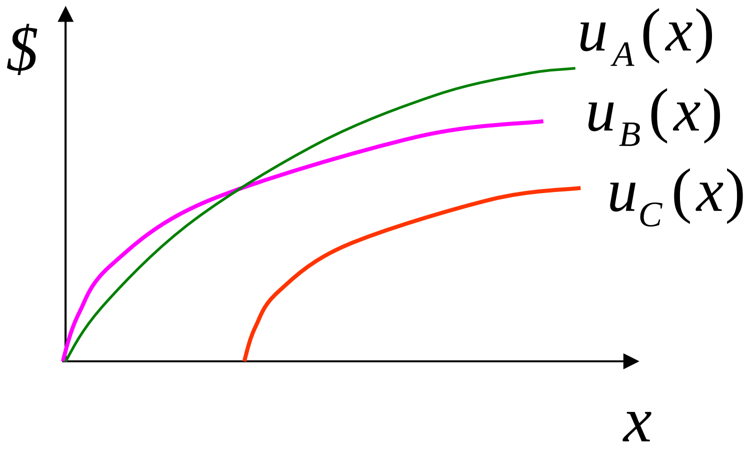
$$p - c + \frac{p - c}{r} - d = 0, \quad \longleftarrow \text{ new entrant balances costs}$$

$$\Rightarrow p_1 - c + \frac{p_1 - c}{r} = s \Leftrightarrow p_1 = c + \frac{r}{1+r}s$$

Example: Pricing in communication networks

The utility function

- Consumers are characterized by the utility function $u(x)$
  - translate into monetary units the benefit of the consumer from the use of the particular network resource
  - has the meaning of trading, reselling

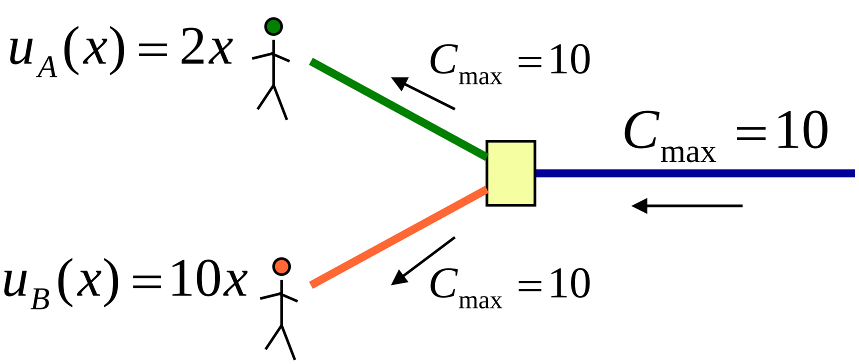


$$u_A(10) = 5, u_B(10) = 2$$

$$u(x_1, x_2)$$

Pricing

- Types of charging:
  - fixed charge: connection cost
  - variable charge: cost related with the size of consumption
  - fixed + variable part
- Variable part: recovery of usage cost, control mechanism (of priority) of consumer



Cost = 1$/unit

Connection Cost = 5$

Cost based charging: $5 + x$

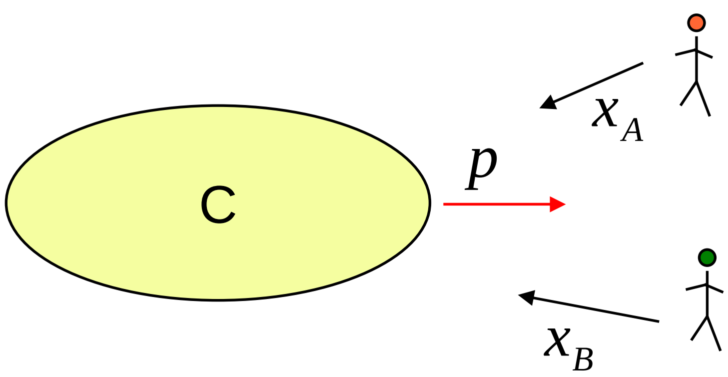
Every user receives $x = C_{\max} / 2 = 5$

Is it economically fair?

Pricing as a control mechanism

- Service provider does not know the utility function of the consumers
- The consumers are looking for their own benefit
- The quantity of the available service is finite
- How can the total benefit of the consumers be maximized? The network profit?
- **Price Mechanism!**

How much should I ask if the price is *p* ;

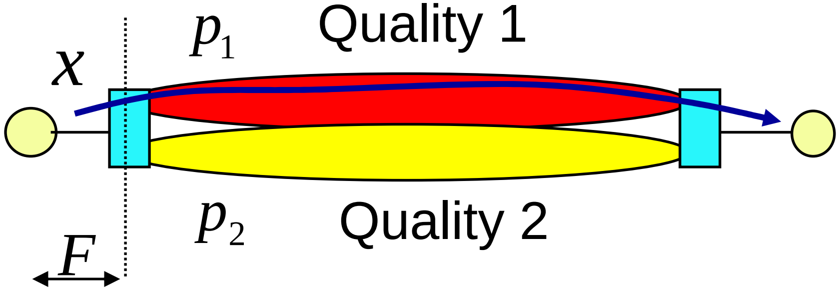


How the problem is specialized for networks;

A possible analysis of a charge

- In general we can analyze the total charge the user is paying as

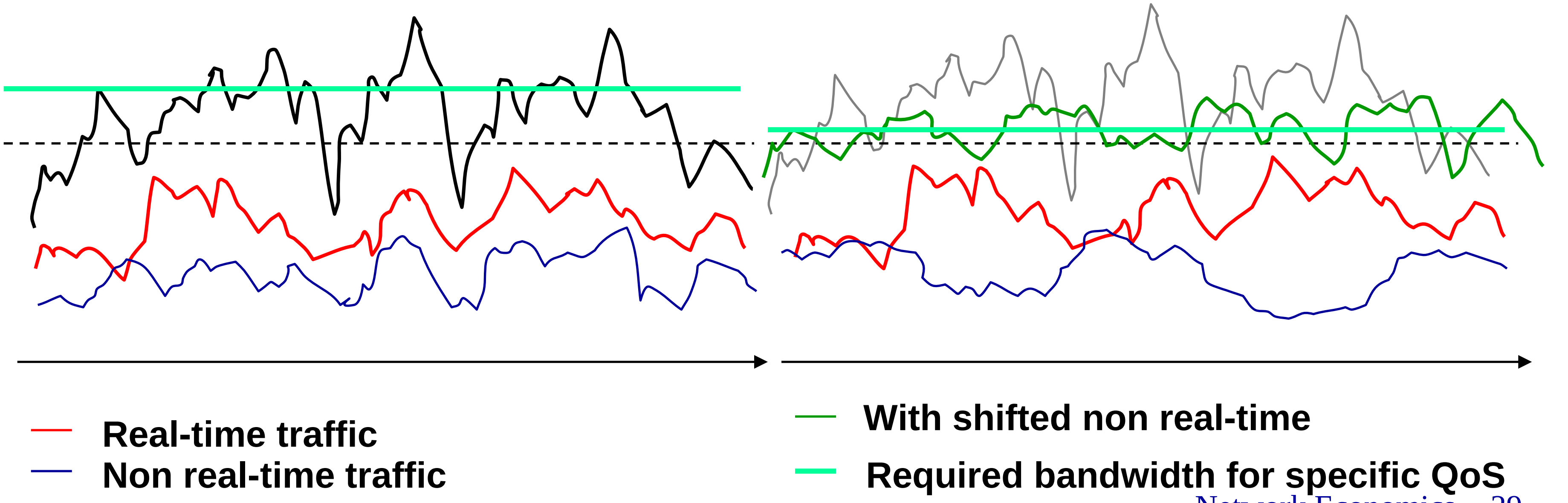
S = F+U+G+Q, where
F= fixed part,
U= usage part,
G= congestion part,
Q= quality part



$$S = F + p_1 xT = F + p_1 q(T)$$

Traffic in the Internet

- Traffic shaping:
  - traffic = real-time + non real-time
  - delay increase => smaller peak rate
  - small delay in non real-time => big difference for the network!
  - Incentives for traffic shaping, priorities



A bandwidth market

- One link, bandwidth = C , two classes of traffic
- Maximization problem:

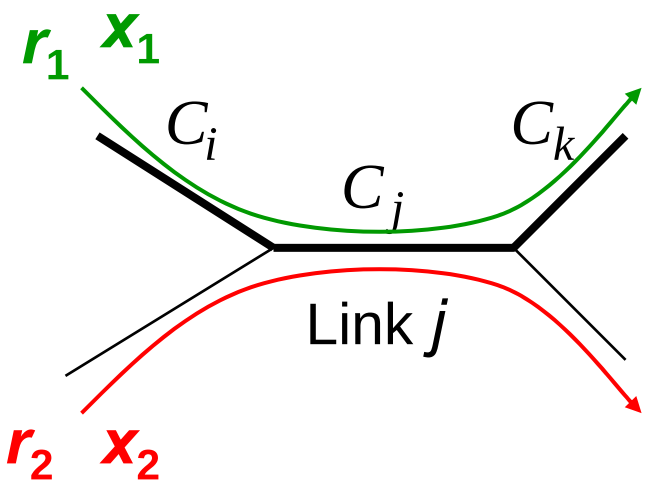
$$\max_{\{x_i^I, x_i^{II}\}} \sum_i u_i(x_i^I, x_i^{II}) \quad s.t.$$

$$\sum_i x_i^I \le \rho_I C_I, \sum_i x_i^{II} \le \rho_{II} C_{II},$$

$$\rho_I < \rho_{II} < 1$$

- Solution: different prices for high and low priority traffic

A network model



System problem

$$\max_{\{x_r\}} \sum_r U_r(x_r)$$

$$s.t. \quad Ax \le C, \quad x \ge 0$$

NETWORK(A,C;*x*)

$$\dot{p}_j = sign(\sum_{r \in j} x_r - C_j)$$

$$\lambda_r = \sum_{j \in r} p_j$$

USERr(*U*$_r$;*λ*$_r$)

$$\max_{x_r \ge 0} U_r(x_r) - \lambda_r x_r$$

x_r →

← λ_r

A decomposition

$$\max_{\{x_r\}} \sum_r U_r(x_r)$$

$$s.t. \quad Ax \le C, \quad x \ge 0$$

SYSTEM(U,A,C)

NETWORK(A,C;*w*)

USERr(*U*$_r$;*λ*$_r$)

$$\max_{w_r \ge 0} U_r\left(\frac{w_r}{\lambda_r}\right) - w_r$$

$$\max_x U_r(x) - \lambda_r x$$

$$\xrightarrow{w_r}$$

$$\lambda_r \longleftarrow x_r$$

$$\lambda_r = w_r / x_r$$

$$\max_{\{x_r\}} \sum_r w_r \log x_r$$

$$s.t. \quad Ax \le C$$

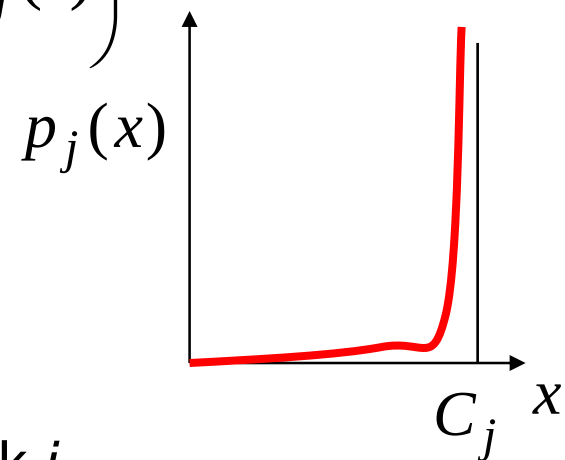
Proportional fairness

Primal algorithm

PRIMAL:

$$\frac{d}{dt} x_r(t) = k\left(w_r - x_r(t) \sum_{j \in r} \mu_j(t) \right)$$

$$\mu_j(t) \quad = p_j\left(\sum_{s: j \in s} x_s(t) \right)$$

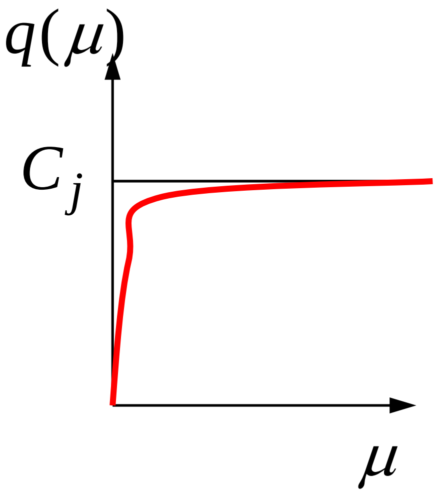


- p_j = marginal congestion cost at link *j*
- μ_j = rate of congestion signals generated at link *j*
- $x(t)$ = multiplicative decrease, linear increase = TCP!
- demo

Dual algorithm

DUAL: $$\frac{d}{dt}\mu_j(t) = k\left(\sum_{r:j\in r} x_r(t) - q_j(\mu_j(t))\right)$$

$$x_r(t) = \frac{w_r}{\sum_{k\in r}\mu_k(t)}$$



- $q_j(\mu_j)$ = flow through resource *j* that generates price μ_j
- $\mu_j(t)$ proportional to excess demand at these prices

Dimensioning of the network

- Prices at the equilibrium can play the role of “signals” for increase or decrease of the required network resources

If $$V(C) = \max_{\{x_i\}} \sum_i u_i(x_i) \quad s.t. \quad \sum_i x_i \le C$$

with Lagrange multiplier λ,

then $$\frac{\partial V(C)}{\partial C} = \lambda$$

So if the marginal cost of C increase is MC,

$$\lambda > MC \Rightarrow \text{increase of } C$$

$$\lambda < MC \Rightarrow \text{decrease of } C$$

Important: the value of λ equals with the equilibrium price in the market

Externalities

Externalities

- Externalities: the actions of one agent affect the utility of an other agent
- Positive (network effects), negative (congestion)
- No externality:

$$\pi_1 = \max_{x_1} u_1(x_1)$$

$$\pi_2 = \max_{x_2} u_2(x_2)$$

$$x_1 + x_2 \le C$$

- Externality:

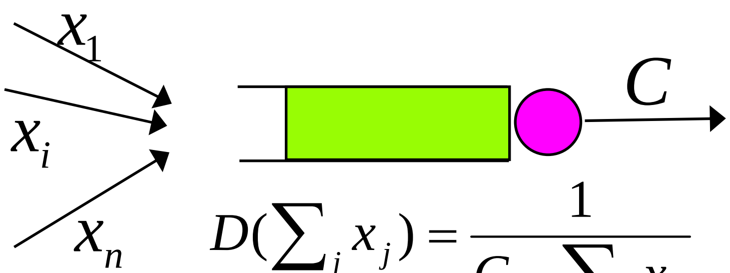
$$\pi_1 = \max_{x_1} u_1(x_1) \pm g_1(x_2)$$

$$\pi_2 = \max_{x_2} u_2(x_2)$$

$$x_1 + x_2 \le C$$

- SW optimal prices can not be determined by the market alone: need special price mechanism that takes account of the externalities

Congestion prices



User i : $u_i(x_i) - \gamma_i x_i D(\sum_k x_k)$

$$D(\sum_j x_j) = \frac{1}{C - \sum_j x_j}$$

$$\text{Max SW} : \max_{x_1,\ldots x_n} \sum_i [u_i(x_i) - \gamma_i x_i D(\sum_k x_k)]$$

$$\Leftrightarrow u_i' - \gamma_i D - \gamma_i x_i D' - D' \sum_{j \neq i} \gamma_j x_j = 0 \qquad (1)$$

$$\text{Free market equilib.} : \text{User i} : \max_{x_i} [u_i(x_i) - \gamma_i x_i D(\sum_k x_k)]$$

$$\Leftrightarrow u_i' - \gamma_i D - \gamma_i x_i D' = 0 \qquad (2)$$

the system is more congested!

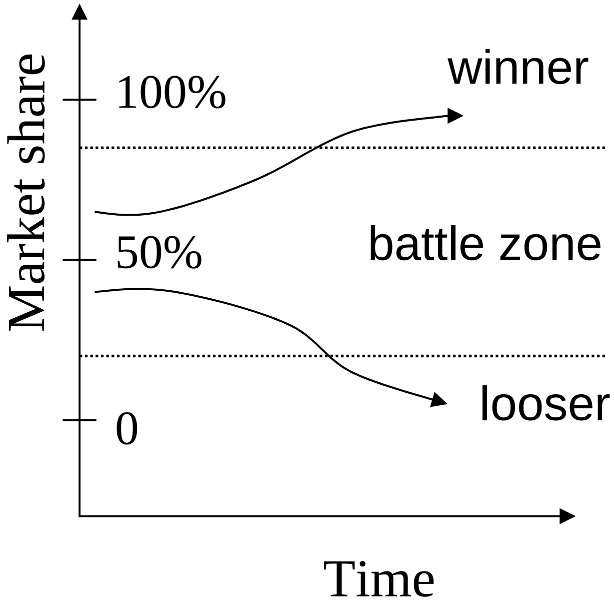
To maximize SW : charge x_i with price $p_i^c = D' \sum_{j \neq i} \gamma_j x_j$

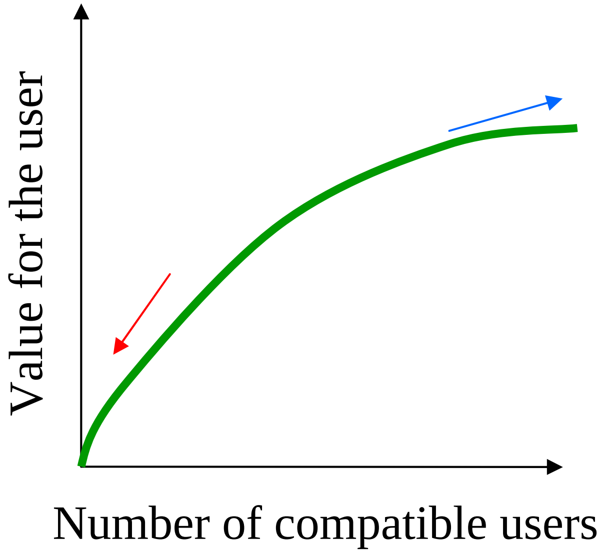
$$\text{User i} : \max_{x_i} [u_i(x_i) - \gamma_i x_i D(\sum_k x_k) - p_i^c x_i]$$

$$\Leftrightarrow u_i' - \gamma_i D - \gamma_i x_i D' - p_i^c = 0 \qquad (3)$$

Externalities and demand

- Positive feedback: strong get stronger, weak get weaker
- Makes a market “tippy”, “winner take all markets”
- Ethernet vs Token Ring, IP vs ATM, Wintel vs Apple
- Number of users is important: Metcalfe’s Law:
  Value of network of size n proportional to n^2





Sources of positive feedback

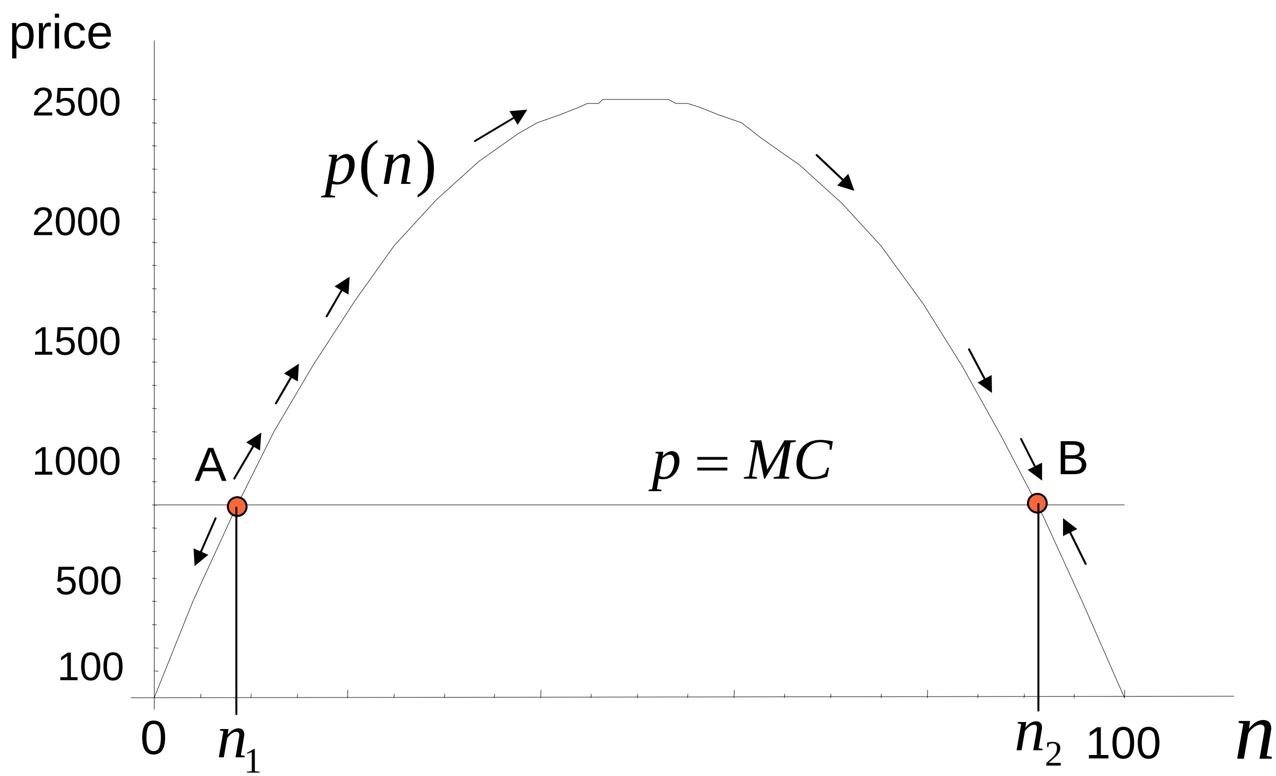
- **Supply side** economies of scale
  - Declining average cost
  - Marginal cost less than average cost
  - Example: information goods
- **Demand side** economies of scale
  - Network effects: virtual networks
    - Network externalities: one market participant affects others without compensation being paid.
  - Examples: telephony, fax, email, Web, Broadband Access, etc.

Network effects

$$i = 1, \ldots, N,\ u_i(n) = ni$$

$$\text{assume } p \to n : N - n + 1, \ldots N \text{ consume}$$

$$\text{marginal customer} = N - n,\ \hat{u} = (N - n)n, \Rightarrow p = n(N - n)$$



Public goods

- Non-excludable and non-rival goods
- Incentive problem in provisioning: the free-rider problem

Example: provision a common facility of size = 1,2

$$u_i(1) = 2,\ u_i(2) = 4,\ c_i(1) = 3$$

Player B

		provision 1	provision 0
Player A	provision 1	4-3,4-3	2-3,2
	provision 0	2,2-3	0,0

Free-riding: player i prefers the other player to contribute
Free-market fails to provision optimum amount of public goods

Peer-to-peer **(joint work with P. Antoniadis and R. Mason)**

- An other case where externalities (positive and negative) are important, public good aspects
- many equilibria, most of them inefficient

Example: two users share files

$$u_i(1) = 2, u_i(2) = 3, c_1(1) = 1.5, c_2(1) = 1.9$$

Player B

Player A		provision 1	provision 0
	provision 1	1.5,1.1	.5,2
	provision 0	2,.1	0,0

Free-riding: player i prefers the other player to contribute
Two equilibria: one is more inefficient than the other

Economic modeling

- Basic model: public good provision, congestion
  - all peers benefit from the contribution of any single peer
  - but contribution is costly
  - positive externality creates an incentive to free-ride on efforts of others
  - consumption causes negative externalities
  - total effect: under-provision

Solutions ?

- One solution: government provision (e.g., national defense)

- For private provision: 2 problems
  - providing incentives to prevent free-riding
  - providing incentives to get information to prevent free-riding (mechanism design)

An economic model of peering

Net utility of peer *i* : $u_i(r, f) = b_i(r_i, \sum_{j=1}^{N} f_j) - c_i(f_i, \sum_{j=1}^{N} r_j)$

Equilibrium strategy: each peer solves

$$\max_{r_i, f_i} b_i(r_i, \sum_{j=1}^{N} f_j) - c_i(f_i, \sum_{j=1}^{N} r_j) \Rightarrow f_i \approx 0$$

r_i : resource request rate of peer *i*

f_i : amount of resources contributed by peer *i*

How do we achieve efficiency? (1)

- Provide incentives
  - different approaches depending on available information
- Case A: Complete information
  - traditional approach: use Lindahl prices
  - A Lindahl price represents total externality imposed by an individual peer
  - hence it is personalized
  - can achieve full efficiency with these prices
  - Prices may be replaced with simple linear rules

Maximizing efficiency

Global planner solves:

$$S \equiv \max_{\{r_i\},\{f_i\}} \sum_{i=1}^{N} \left[b_i(r_i, \sum_{j=1}^{N} f_j) - c_i(f_i, \sum_{j=1}^{N} r_j) \right] \longrightarrow \{r_i^*, f_i^*\}$$

Use prices: find p_i^r, p_i^f so that peer *i* chooses r_i^*, f_i^*

$$\max_{r_i, f_i} \left[b_i(r_i, f^*) - p_i^r r_i + p_i^f f_i - c_i(r^*, f_i) \right] \longrightarrow r_i^*, f_i^*$$

Use rules: find α_i, β_i so that peer *i* chooses r_i^*, f_i^*

$$\max_{r_i, f_i} \left[b_i(r_i, f^*) - c_i(r^*, f_i) \right] \quad s.t.\ r_i \le \beta_i f_i + a_i \longrightarrow r_i^*, f_i^*$$

2 problems with Lindahl prices

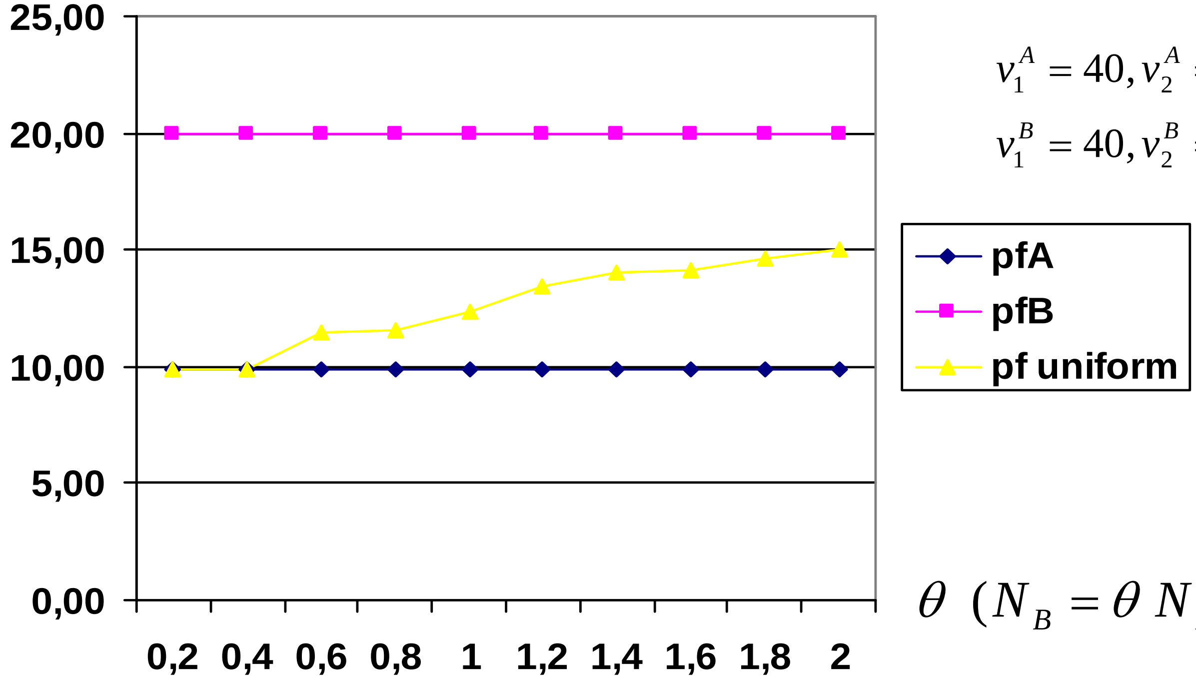
- informationally very demanding (complete information)
- this can be relaxed in a large network: personalized prices can be approximated by a uniform price
- payments present difficulties in a large, anonymous network with many small transactions
- Use rules instead of prices

Interesting results

- For large N uniform prices, but not uniform rules
- Stability of rules
- Practical perspective
  - heuristics to approximate optimal prices and rules for mixed groups using information from single-type groups

Heuristics

The uniform price of the mixed group depending on N_A, N_B



$v_1^A = 40, v_2^A = 40, k_1^A = 5, k_2^A = 0.1$

$v_1^B = 40, v_2^B = 40, k_1^B = 10, k_2^B = 0.1$

$\theta \ (N_B = \theta \ N_A)$

Simple example: $b_i = v_1 \log r_i + v_2 \log \sum_j f_j - k_1 f_i - k_2 \left(\frac{f_i}{\sum_j f_j} r_j\right)^2$, 2 types (A and B)

How do we achieve efficiency? (2)

- Case B: incomplete information
  - model situation as a Bayesian game
  - peers know the distribution of the benefit and cost of other peers
- 2 types of inefficiency
- typically there are many equilibria, distinguished by who contributes
  - all equilibria are inefficient: free-riding is systematic
  - some equilibria are more inefficient than others: may have the ‘wrong’ peers contributing

The effect of heterogeneity

- Result: if peers are very different, 2nd inefficiency is not important: heterogeneity leads to a unique “good ” equilibrium (peers that value the shared resource most contribute most)
  - study of Gnutella shows that bandwidth, latency, availability and degree of sharing vary across peers by 3--5 orders of magnitude

Avoiding free-riding

- Mechanism design
  - model explicitly peers' private information
  - give peers incentives to behave truthfully (incentive compatibility) …
  - … and to join network (participation)
  - …and to contribute resources (cost coverage)
  - typically, full efficiency cannot be attained
- 2 problems with this approach
  - payments still necessary (to give informational incentives)
  - best mechanisms can be very complex and require large amounts of information to be collected centrally

Result: as the network becomes large

- simple model for peer i

$$u_i(f_i, \sum_j f_j) = \theta_i u(\sum_j f_j) - c_i f_i$$

- best mechanism may become very simple: minimum contribution specified for each peer
- in certain circumstances, same contribution can be set for all peers
- in less restrictive cases, contributions have to be set for identifiable groups of peers

Information

- Economic agents that interact make decisions based on information available regarding the other agents
- Less information available leads to decrease of efficiency
- **Adverse selection** occurs when some type of agent finds it profitable to choose an offer intended for another type. As a result, the seller obtains less profit than anticipated
  - There may be no prices for firm to recover costs
  - $\Rightarrow$ no equilibrium
  - Beneficial for both seller and buyers to **signal** information

Adverse selection and ISPs (1)

- n potential customers, each requiring x units of Internet use, x *uniformly* distributed on [0,1]
- A customer of type x has a utility $u(x) = x \Rightarrow$ he won't buy service if his *surplus* $x - w$ is negative
- The network exhibits economies of scale. The **unit cost** when using total bandwidth b for its customers is $p(b) \leq 1$
  - $p(b)$ includes a **discount factor** that varies linearly from $\alpha < 1$ to 1 with the total amount of bandwidth purchased

$$p(b) = a \frac{b}{n/2} \quad + 1 \quad \left(1 - \frac{b}{n/2}\right)$$

Adverse selection and ISPs (2)

- **Complete information**:
  - customer of type x is charged $w(x) = x - \varepsilon$
- All customers subscribe, provider and customers have positive profits

$$p(n/2) = \alpha < 1$$

$\pi(x) = x - \varepsilon - x\alpha = x(1-\alpha) - \varepsilon > 0$ for small enough ε

Adverse selection and ISPs (3)

- **Incomplete information**: price is same for all customers
- **Adverse selection**: price targeted to recover costs for average customer, heavy customers profit and increase average cost => no stable market
- Assume that provider charges w
- $n(1-w)$ heaviest customers subscribe, $b = 1/2n(1-w)(1+w)$
- Typical customer $\bar{x} = 1/2(1+w)$
- Profit from typical customer =

$$\bar{\pi} = w - \frac{1}{2} p(b)(1+w) = w - \frac{1}{2}[1-(1-w^2)]\left(1+w\right)$$

$\boxed{\bar{\pi} < 0 \quad \text{if} \quad \alpha > 0.7465 \quad \text{for all values of} \quad w}$

Marginal cost pricing and cost recovery

Marginal cost prices

- **Strong points:**
  - welfare maximisation under appropriate conditions
  - firmly based on costs
  - easy to understand
- **Weak points:**
  - **do not cover total cost** (need for subsidisation)
  - must be defined w.r.t. time frame of output expansion?
    - **short run marginal cost = 0 or ∞**
    - use long-run marginal cost (planned permanent expansion)
  - difficult to predict demand and to dimension the network
  - difficult to relate cost changes to marginal output changes

Marginal cost pricing (cont.)

- Marginal cost = covers all sacrifices, present or future, external or internal to the company, for which production is at **the margin** **causally** responsible
- Problem1: **specifying the time perspective**
  - should we use long-run MC rather than short-run MC?
  - MC includes present and future causally attributed costs
  - problem: total cost coverage
- Problem2: **specifying the incremental block of output**
  - incremental cost depends on size of increment
  - charge the shortest run MC for the smallest output increment
- Problem3: **large proportions of common costs**

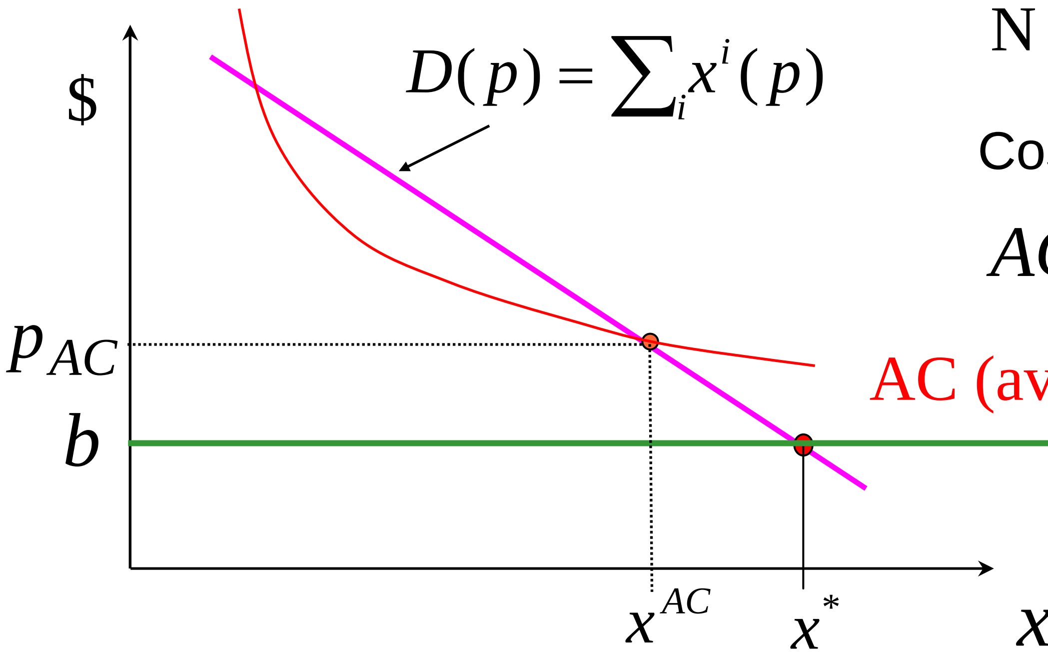
Recovering network cost

- **Pricing at marginal cost maximises efficiency but does not necessarily recover network cost**
  - example: assume $c(x) = \alpha + \beta x$

  Then under marginal cost pricing, $p = \beta$

  and the network revenue is βx, hence we are short of α
- **Ways out:**
  - add fixed fee (two-part tariffs)
  - Ramsey prices
  - general non-linear tariffs

Two-part tariffs



N customers

Cost = $a + bx$

$$AC = a/x + b$$

Under *MC* pricing, network needs to recover an additional amount a

Use tariff $a/N + bx$

Customer benefit = $u(x(b)) - a/N - bx(b)$ **< 0 ?**

$x(b) =$ user demand at price b