Tunable Fermi gases experiments in the BEC-BCS crossover


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D. Petrov, G. Shlyapnikov, R. Combescot, Y. Castin,
Fermi superfluid and Bose-Einstein condensate of Molecules

Fermions with two spin states with attractive interaction

BEC of molecules \[ \rightarrow \] BCS fermionic superfluid

Bound state \[ \leftarrow \] Interaction strength
No bound state

Leggett, Eagles, Nozières, Schmidt-Rink,… ’80

Dilute gases: Feshbach resonance
Outline

General methods for ultracold Fermi gas manipulation

Tuning the interaction in the gas

Molecule formation and Bose-Einstein condensation of fermion dimers

Crossover experiments and superfluidity

Superfluidity with spin population imbalance

Prospects
Quantum statistics in harmonic traps

- **Bose-Einstein statistics (1924)**
  - Bose-Einstein condensate
  - Bose enhancement
  - \( T_c = \frac{\hbar \omega}{k_B} (0.83 \ N)^{1/3} \)
  - Dilute gases: 1995, JILA, MIT

- **Fermi-Dirac statistics (1926)**
  - Fermi sea
  - Pauli Exclusion
  - \( T \ll T_F = \frac{\hbar \omega}{k_B} (6 \ N)^{1/3} \)
  - Dilute gases: 1999, JILA
Collisions between identical particles and quantum statistics

Direct Exchange
Scattering amplitude interfere with + sign for bosons and – for fermions

At low temperature, s-wave only

\[ |\psi_f> = \frac{1}{\sqrt{2}} (1 + \varepsilon P_{21}) |1: k e_z, 2: -k e_z> \]

Scattering amplitude interfere with + sign for bosons and – for fermions

At low temperature, s-wave only

Bosons
\[ \sigma = 8\pi a^2 \]

Fermions
\[ \sigma = 0 \]
Good for clocks: no interaction shift
But evaporation is more difficult
Sympathetic cooling
**Solution 1:** sympathetic cooling (with bosons)

\[ ^{6}\text{Li}-^{7}\text{Li}, \quad ^{6}\text{Li}-^{23}\text{Na}, \quad ^{6}\text{Li}-^{87}\text{Rb}, \quad ^{40}\text{K}-^{87}\text{Rb}, \quad \ldots \]

**Solution 2:** mixture of spin states in magnetic trap

**Solution 3:** mixture of spin states in optical trap + Feshbach resonance

<table>
<thead>
<tr>
<th>Location</th>
<th>Year</th>
<th>Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>JILA Boulder</td>
<td>1999</td>
<td>magnetic trap, spin mixture</td>
</tr>
<tr>
<td>LENS Florence</td>
<td>2002</td>
<td>magn. trap &amp; sympathetic cooling Rb</td>
</tr>
<tr>
<td>ETH Zurich</td>
<td>2004</td>
<td>magn. trap &amp; sympath. cooling Rb</td>
</tr>
<tr>
<td>Univ. Hamburg</td>
<td>2005</td>
<td>magn. trap &amp; sympathtic cooling Rb</td>
</tr>
<tr>
<td>Univ Toronto</td>
<td>2005</td>
<td>chip magn. trap &amp; sympath. cooling Rb</td>
</tr>
<tr>
<td>Rice University</td>
<td>2001</td>
<td>magn. trap, sympathetic cooling $^{7}\text{Li}$</td>
</tr>
<tr>
<td>ENS Paris</td>
<td>2001</td>
<td>magn. trap, sympathetic cooling $^{7}\text{Li}$</td>
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<tr>
<td>Duke University</td>
<td>2001</td>
<td>optical dipole trap, mixt.of spin states</td>
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<td>MIT Boston</td>
<td>2002</td>
<td>magn. trap, sympathetic cooling $^{23}\text{Na}$</td>
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<td>Uni. Innsbruck</td>
<td>2003</td>
<td>optical dipole trap, mixt.of spin states</td>
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<td>Univ. Tubingen</td>
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<td>chip magn. trap, sympathetic cooling Rb</td>
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<tr>
<td>Uni. Swinburne</td>
<td>2007</td>
<td>optical dipole trap, mixt.of spin states</td>
</tr>
<tr>
<td>Univ. Kyoto</td>
<td>2006</td>
<td>optical dipole trap, mixt.of spin states</td>
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</tbody>
</table>
Bose-Einstein condensate and Fermi sea

10^4 Li\textsubscript{7} atoms, in thermal equilibrium with 10^4 Li\textsubscript{6} atoms in a Fermi sea.

Quantum degeneracy: T = 0.28 \mu K = 0.2(1) T\textsubscript{C} = 0.2 T\textsubscript{F}

F. Schreck et al., PRL 01
Use Sodium F = 2 as refrigerator to cool Lithium in F = 3/2 (state $|\psi\rangle$) in a magnetic trap. 20s forced evaporation on Na results in typically $10^7$ atoms in BEC (w/o Li) and $50 \times 10^6$ Li atoms at $T/T_F < 0.3$.

All-optical method

All-optical approach by John Thomas group at Duke Univ., Durham, NC, USA

**CO$_2$ laser trap**

- Beam waist: $2w = 2 \times 47 \mu$m
- Rayleigh range: $2z_R = 2 \times 660 \mu$m
- Trap depth: 690$\mu$K

Loading of a few $10^6$ atoms *directly from the MOT*

After plain evaporation for 5s:
- $1.3 \times 10^6$ atoms @ 5$\mu$K, p.s.d. $8 \times 10^{-3}$ ($T/T_F = 2.8$)
- Now $2 \times 10^5$ atoms at $T/T_F = 0.1$

Also Innsbruck Univ.

*ultrastable CO$_2$ trapping of lithium fermions*

O’Hara et al., PRL 82, 4204 (1999)
Optical Traps

Dipole force: far-off resonance laser: very low photon scattering rate
Flexible geometry, 1 or several beams, adjustable aspect ratio

Decouples trapping function and magnetic tuning for Feshbach Resonances

Can be switched on and off very fast

Easy modulation of trap depth or position using acousto-optic modulators
Excitation of collectives modes, rotating trap,….

3D, 2D, 1D optical lattices by interference of several laser beams

See e.g. Proceedings of 2006 Varenna School on Cold Fermi Gases on cond-mat, and book to appear in 2007
And R. Grimm, Y. Ovchinikov ‘00
BCS theory: Cooper pairs

Bardeen, Cooper, Schrieffer, 1957

Superconductivity of metals at low temperature

Homogeneous Fermi gas, $k_F$, $E_F$ at zero temperature

Add two fermions, 1 and 2 with different spin states, with attractive interaction: $a_{\uparrow\downarrow} < 0$

$$V(\vec{r}_1 - \vec{r}_2) = V\delta(\vec{r}_1 - \vec{r}_2) \quad \text{with} \quad V < 0$$

The state with correlated pairs of fermions has energy lower than $E_F$.

$$\vec{k}, -\vec{k} \quad \text{Pairs at Fermi surface:} \quad |k| \geq k_F$$

If $T$ is low enough, a superfluid phase is produced

Critical temperature:

$$T_{BCS} \sim 0.28 \, T_F \, e^{-\frac{\pi}{2k_F|a|^{\uparrow\downarrow}}}$$

Validity: $k_F a << 1$

Example: $k_F a = -0.2$, $T_{BCS} = 10^{-4} \, T_F$ is very small!!
Tuning atom-atom interactions
$^6$Li Ground state in magnetic field
\( E_B = -\frac{\hbar^2}{ma^2} \)

Lithium 6 Feshbach resonance
BCS phase condensate of molecules

BEC-BCS Crossover

scattering length [nm]

Magnetic field [kG]

Bound state

\[ Eb = \frac{\hbar^2}{ma^2} \]

condensate of molecules

No bound state

BCS phase
Experimental approach

Cooling of $^7\text{Li}$ and $^6\text{Li}$

1000 K: oven

1 mK: laser cooling

10 $\mu$K: evaporative cooling
   in magnetic trap

$E = -\mu \cdot \vec{B} = +|\mu| |\vec{B}|$

Tuning the interactions in optical trap

Final evaporation in optical trap
Evaporation of $^6$Li gas in an optical trap

Two YAG beams with 2.5 W and waist of 38 $\mu$m

Temperature is measured in the weakly interacting regime ($B < 200$ G) by fit to the finite T Fermi distribution.

Difficult to get T in the crossover region (except in imbalanced case, MIT).

Thermal fraction on molec BEC side, or universal thermodynamics at unitarity.

$T_F = 5 \, \mu K$

$T/T_F = 0.2$

$N_{total} = 1 \times 10^5$
**Recipe:** in region $a<0$, cool a gas of fermions below $T_F$
Slowly scan across resonance towards $a>0$
Typically: 1000 G to 770 G in 200 ms
This produces molecules with up to 90% efficiency!
Reversible process! Entropy is conserved.
If $T<0.2 \ T_F$, BEC of molecules
Condensates of molecules

in situ

$^6\text{Li}_2$

$^6\text{Li}_2$

Expansion


Innsbruck Bartenstein et al., PRL 92, 120401 (2004)

MIT Zwierlein et al., PRL 91, 250401 (2003)

ENS Bourdel et al., PRL 93, 050401 (2004)

Rice Partridge et al., PRL 93, 020404 (2005)

2007: Swinburne Univ
A simple thermodynamic model

conservation of entropy

See also T. Koehler lecture
BEC of molecules: excellent starting point for exploring the crossover

Q1: Lifetime of molecules?

Q2: Interaction between molecules?

Q3: What happens in strongly correlated regime: unitarity: $k_Fa >> 1$?

Q4: Can we measure the excitation gap?

Q5: How to probe superfluidity in crossover regime?

Q6: What is the momentum distribution of particles?

Q7: Superfluidity with imbalanced spin populations?
Remarkable stability of weakly bound molecules
Suppression of vibrational relaxation for fermion dimers

Binding energy: $E_B = \frac{h^2}{ma^2}$
Momentum of each atom: $\frac{h}{a}$

Pauli exclusion principle
Inhibition by factor $(a/R_e)^2 >> 1$

G~ $\frac{1}{a^s}$ with $s = 2.55$ for dimer-dimer coll.
3.33 for dimer-atom coll.

D. Petrov, C.S., G. Shlyapnikov, PRL 04
Comparison with experiments

\[ \beta_{\text{exp}} \sim a^{-2.3 \pm 0.4} \]

\[ \beta_{\text{th}} \sim a^{-2.55} \]

On resonance, lifetime of strongly interacting gas exceeds 30 s!
Interaction between molecules measurement of $a_{mm}$

nearly pure condensate $\lambda = 0.1$

Hydrodynamic expansion is signature of superfluidity on BEC side

$T \leq 0.9 \mu K = T_c^0 / 3$

In trap TF radius:

$R_x = 26 \mu m, R_y = 2.75 \mu m$

Good agreement with theory:

$a_{mm} = 0.6 a = 0.6 \times 306 = 183 \text{ nm}$

D. Petrov, G. Shlyapnikov, C.S.

Excludes: $a_{mm} = 2a$

From hydrodynamic expansion

At 770 G: $a_{mm} = 170^{+100}_{-60} \text{ nm}$
Prepare nearly pure condensate at 770G: $4 \times 10^4$ mol., $N_0/N \geq 70\%$

Change magnetic field slowly across FR: rate: 1-2 G/ms

Take 1.4 ms TOF image

Trap aspect ratio: $\lambda = 0.3$

Resonance position 823 G

$a < 0$

$a > 0$
On resonance $k_F a >> 1$, behavior should no longer depend on $a$. Equation of state should have same density dependence as ideal Fermi gas.

\[
\mu = \frac{\hbar^2}{2m} (6\pi^2)^{2/3} (1 + \beta)n^{2/3}
\]

$\beta = 0$: ideal Fermi gas

$\beta \neq 0$ at unitarity

On resonance: $E_R = \sqrt{1 + \beta} E_R^0$

Where $E_R^0$ is the release energy of non-interacting Fermi gas in harm. trap

We find: $\beta = -0.58(15)$

A fundamental quantity in many-body theories

Good agreement with QMC method (Carlson 02 Giorgini 04, UMASS-ETH coll. 05)

T. Bourdel et al., PRL 2004
Universal equation of state of Fermi gas with equal spin populations

balanced Fermi gas ($\mu_{\uparrow} = \mu_{\downarrow}$)

$$n = \frac{1}{6\pi^2} \left( \frac{2m\mu_{\uparrow}}{\hbar^2} \right)^{3/2} \times \text{numerical factor}$$

$$\mu_{\uparrow} = \xi \frac{\hbar^2}{2m} \left( 6\pi^2 n \right)^{2/3} = \xi E_F$$

Determination of $\xi$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENS ($^6$Li)</td>
<td>0.42(15)</td>
</tr>
<tr>
<td>Rice ($^6$Li)</td>
<td>0.46(5)</td>
</tr>
<tr>
<td>JILA ($^{40}$K)</td>
<td>0.46(10)</td>
</tr>
<tr>
<td>Innsbruck ($^6$Li)</td>
<td>0.27(10)</td>
</tr>
<tr>
<td>Duke ($^6$Li)</td>
<td>0.51(4)</td>
</tr>
<tr>
<td>Theory</td>
<td></td>
</tr>
<tr>
<td>BCS</td>
<td>0.59</td>
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<tr>
<td>Astrakharchik</td>
<td>0.42(1)</td>
</tr>
<tr>
<td>Perali</td>
<td>0.455</td>
</tr>
<tr>
<td>Carlson</td>
<td>0.42(1)</td>
</tr>
<tr>
<td>Haussmann</td>
<td>0.36</td>
</tr>
</tbody>
</table>
Aspect ratio at low temperature

On resonance: agreement with hydrodynamic prediction
At Bc: crossing of the critical temperature near 930 Gauss.
For T > Tc, generalized Cooper pairs are broken, hence loss of superfluidity.
At higher T, the step smoothes and shifts towards smaller 1/k_F|a|
Critical temperature in BEC-BCS crossover

At $B_c$, $1/k_Fa = -0.33$ in trap
Phase diagram at T=0

Molecular BEC
Strongly bound
Size: $a \ll n^{-1/3}$
n$^{-1/3}$: average dist. between particles

BCS regime:
$k_F|a| \ll 1$
Cooper pairs $k$, -$k$
Well localized in Momentum: $k \sim k_F$
Delocalized in position

On resonance
$n a^3 > 1$ or $k_F a \geq 1$
Pairs stabilized by Fermi sea

F. Chevy
C.S.
Physics World
March 05
Observation of pairing gap

Innsbruck
C. Chin et al., Science 04

T = 5 T_F

T < 0.2 T_F
T_F = 1.2 μK

On resonance: hΔ ~ 0.2 E_F
Are fermion pairs condensed?

Condensation of fermionic pairs: JILA, MIT

Starting field above resonance: 900 G

Initial temperature: \( T / T_F = 0.2 \)

High condensate fraction indicates the presence of \( k, -k \) pairs on resonance side where no molecular bound state exists.

80% condensed fraction

\[ ^{6}\text{Li} \]
Pair condensation transition temperature: $40\,K$

Theory: Q. Chen et al., PRA 73, 041601 (2006)

Enrico Fermi Summer School 2006
So far:
- Anisotropic expansion
- Collective modes
- Pairing gap
- Condensate fractions

are evidence for superfluid behavior

Direct proof of superfluidity in the system?

Put the gas in rotation
In contrast to classical gas, the superfluid Fermi gas should exhibit quantized vortices, \((\hbar/2m)\) (Sandro Stringari’s lecture)
Observation of vortex lattices in the BEC-BCS crossover (MIT, 05)

Direct proof of superfluidity

M. Zwierlein
A. Schirotzek
C. Stan
C. Schunk
P. Zarth
W. Ketterle
Science 05
Pair breaking in TOF [ms] 930 G
Superfluidity with imbalanced spin populations
Attractive Fermi gas with equal spin population
⇒ BCS theory, pairing at edge of Fermi surface

What is the nature and existence of superfluidity when spin population is imbalanced?
Mismatched density and/or pairing with different masses

Ex:
Superconductors in magnetic field or quark matter
Cold gases: MIT and Rice expt
Overview of Theoretical scenarios

Chandrasekhar and Clogston: stability of the paired state: $\mu_\uparrow > \mu_\downarrow$

Conversion of a particle: $\downarrow \rightarrow \uparrow$
Decrease the grand potential: $H - \mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow : \mu_\uparrow - \mu_\downarrow$
Cost of pair breaking: $\Delta$
$\Rightarrow$ Paired state stable for $\mu_\uparrow - \mu_\downarrow < \Delta$

And beyond?

Polarized phase: One spin species (Carlson, PRL 95, 060401 (2005))

FFLO Phase (Fulde Ferrell Larkin Ovchinikov): pairing in $k_\uparrow - k_\downarrow \neq 0$
(C. Mora et R. Combescot, PRB 71, 214504 (2005))

Sarma phase (internal gap): pairing in $k_\uparrow - k_\downarrow = 0$
Opening of a gap in the Fermi sea of majority species. (Liu, PRL 90, 047002 (2003))
Superfluidity observed in Time of flight
Loss of superfluidity for large Spin population imbalance

MIT experiment
(Science Express, December 22, 2005)
Experimental results

MIT: 3 phases
- Fully paired superfluid core
- Intermediate mixture
- Fully polarized rim


Rice: 2 phases
Fully paired superfluid core
Fully polarized rim


G. Partridge *et al.*, Cond-mat 0608455
\[ P = \frac{(N_1 - N_2)}{(N_1 + N_2)} \]

- \( P = 0 \)
- \( P = 0.18 \)
- \( P = 0.37 \)
- \( P = 0.6 \)
- \( P = 0.79 \)
- \( P = 0.95 \)

Rice Univ: phase separation at unitarity

\[ |1> - |2> \]

\[ R = \text{Rice Univ: phase separation at unitarity} \]

Partridge et al., PRL 97, 190407 (2006)
Avalanche of recent publications!

P. Pieri and G.C. Strinati, cond-mat/0512354: diagrammatic method, Extrapolation from BEC regime
W. Yi and L.-M. Duan, cond-mat/0601006: BCS at finite temperature
M. Haque and H.T.C. Stoof, cond-mat/0601321: BCS at T=0
T.N. de Silva and E.J. Mueller, cond-mat/0601314: BCS at T=0
D. Sheehy, L. Radzihovsky, PRL 06
A. Bulgac, M. McNeil Forbes ’06
K. Levin et al., 06
M. Parish, Nature Physics 3 ’07

F. Chevy approach:
Assumptions:
1) Unitarity: universal parameter $\mu = (1 + \beta)$ $E_F = \xi E_F$ known
2) Grand canonical description, Local density approx,
3) T=0 approach
Universal phase diagram of the homogeneous unitary system (2)

\[ \Omega = -P V \]

\[ dP = \sum_{\sigma = \uparrow, \downarrow} n_\sigma d\mu_\sigma \]

\[ \Rightarrow \quad \text{Just need to know} \quad n(\mu) \]

\[ P = P_0 (1 + \eta)^{5/2} / (2\xi)^{3/2} \]

\[ \eta = \mu_\downarrow / \mu_\uparrow \]

\[ \eta_\alpha > \eta_c > \eta_\beta \]

\[ \eta_c = (2\xi)^{3/5} - 1 \approx -0.099 \]
Theoretical evidence for an intermediate phase

General properties of a mixed branch?

Step 1: calculate the energy $E$ of a single impurity atom immersed in a Fermi sea ($E = \mu_\downarrow$, with $n_\downarrow = 0^+$)
For $a = \infty$, $E = -0.606 E_F$, $\eta_\beta < -0.606 < \eta_c \sim -0.1$
Step 2: $dP/d\mu_\sigma = n > 0$

$\eta_\beta < \eta_c$: the new branch is stable

$\eta_\beta > \eta_c$: the new branch is unstable

Confirmed by Monte-Carlo
C. Lobo et al. 07
Open questions and perspectives

- Imbalance of spin populations, properties of mixed phase ?, phase diagram phase separation, role of trap anisotropy (M. Randeria)

- Single particle excitations by Raman transitions, T.L. Dao et al., PRL 07, 98

- p-wave pairing ?

- Fermions in optical lattices: simulation of condensed matter Hamiltonians

- Fermionic Hubbard model

- $^6$Li: Transition toward antiferromagnetic order: Néel transition

- Lattices with frustration

- Fermi-Fermi mixtures: pairing with different masses

- Bose-Fermi mixtures

\[ T_{\text{Néel}} \sim 30 \text{ nK} \]

F. Werner et al., PRL 05
Thank you for your attention!