Feshbach Resonances in Ultracold Quantum Gases

Thorsten Köhler

Department of Physics and Astronomy, University College London,

Gower Street, London, WC1E 6BT, United Kingdom

Outline

- Interactions in ultracold gases

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- Universal bound states

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- Magnetically-tunable interactions

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- Classification of Feshbach resonances

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- Diatomic Feshbach molecules
- Outlook

Interactions in ultracold gases

Typical parameters

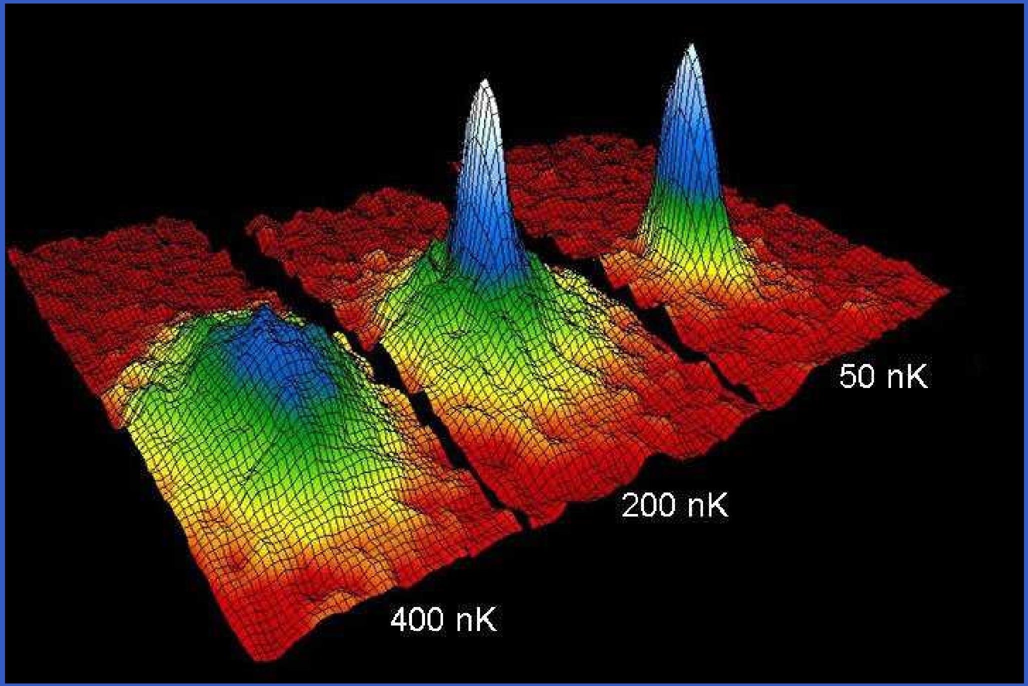
temperature 10 - 100 nK

density 10^{13} - 10^{14} cm^{-3}

number of atoms 10^3 - 10^7

size 10 μm - 1 mm

lifetime 10 s



JILA & MIT 1995

Condensed species

^{87}Rb Na ^{7}Li H ^{85}Rb ^{4}He* ^{41}K Cs Yb Cr ^{39}K

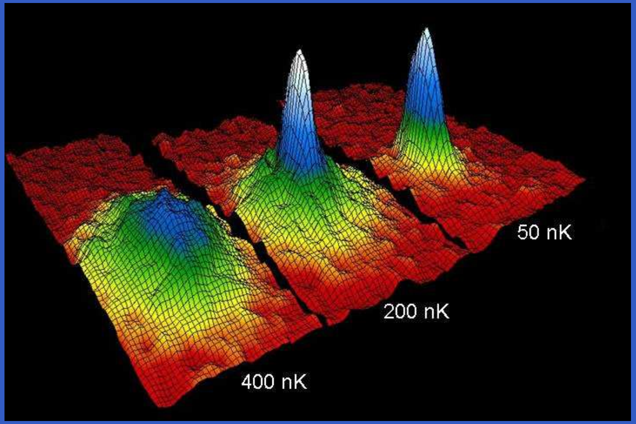
Quantum degenerate fermions ^{40}K ^{6}Li ^{3}He* etc.

Interactions in ultracold gases

How does one describe a Bose-Einstein condensate?

The Gross-Pitaevskii equation for the macroscopically occupied quantum state of each single atom, the **condensate wave function**:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V_{\text{trap}}(\mathbf{x})\right]\Psi(\mathbf{x},t) + \frac{4\pi\hbar^2}{m}a\left|\Psi(\mathbf{x},t)\right|^2\Psi(\mathbf{x},t)$$



E.P. Gross, Nuovo Cimento **20**, 454 (1961)
L.P. Pitaevskii, Sov. Phys. JETP **13**, 451 (1961)
Review: F. Dalfovo, S. Giorgini, L.P. Pitaevskii, and S. Stringari, RMP **71**, 463 (1999)

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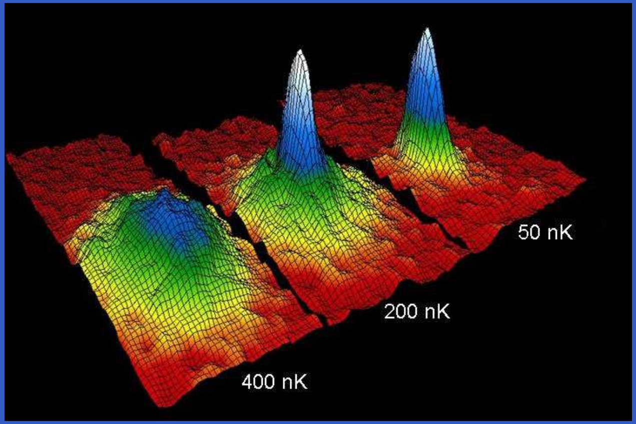
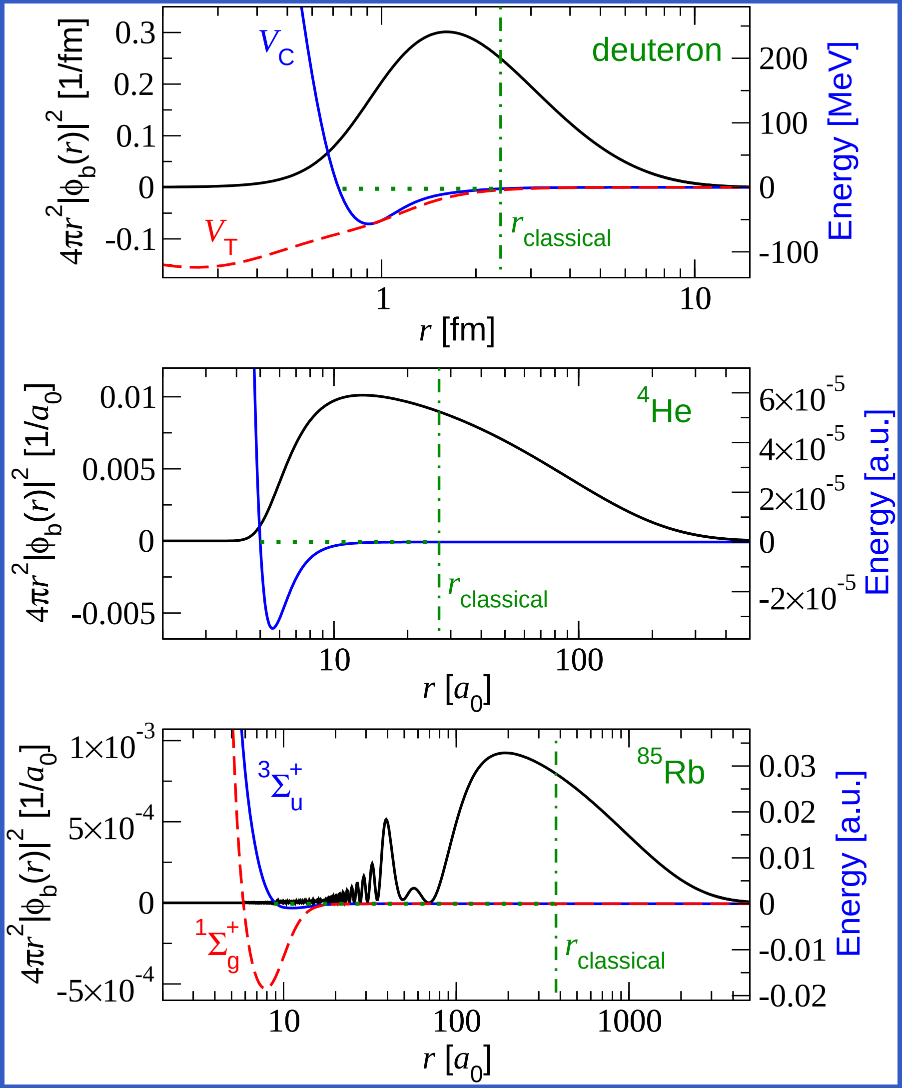


The inter-atomic interactions are characterised solely by the
s**-wave scattering length**, a!

Universal bound states



The average separation between constituent particles exceeds the distance scale set by the classical motion in the potential well!

Classic examples: deuteron and $^4He_2$

New example: Feshbach molecules

Review: A.S. Jensen, K. Riisager, D.V. Fedorov, and E. Garrido, RMP **76**, 215 (2004)

Universal bound states

Universal properties of diatomic halo molecules

- Hamiltonian:

$$H_{2\mathrm{B}} = -\frac{\hbar^2}{m}\boldsymbol{\nabla}^2 + V(r)$$

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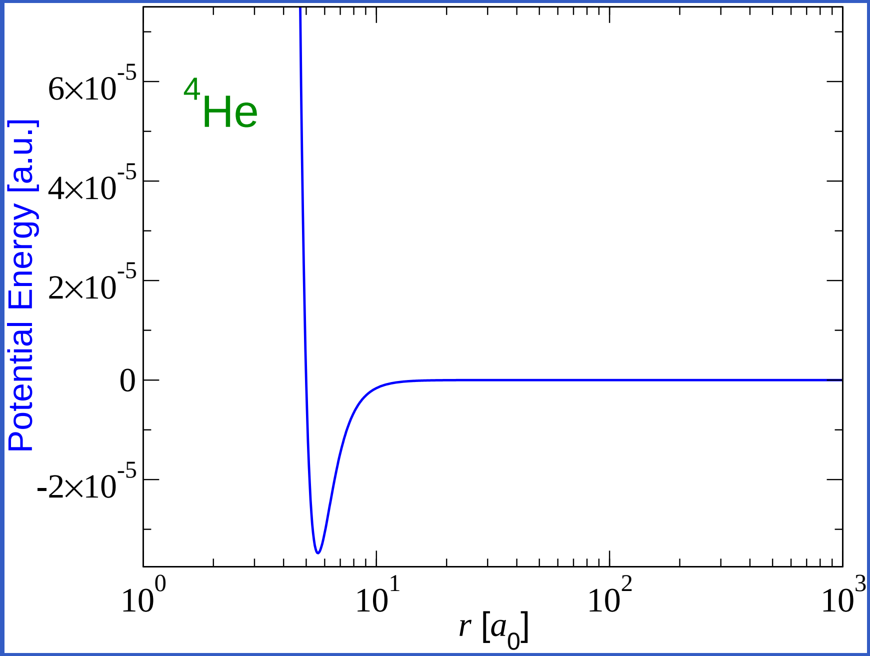

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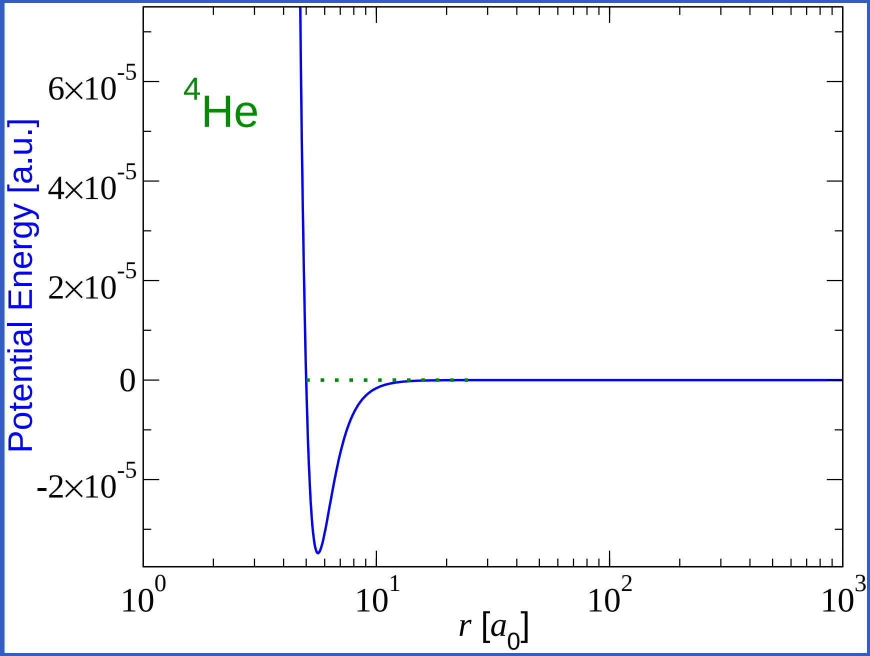
- Bound-state energy:

$$E_b = -\hbar^2/(ma^2)$$



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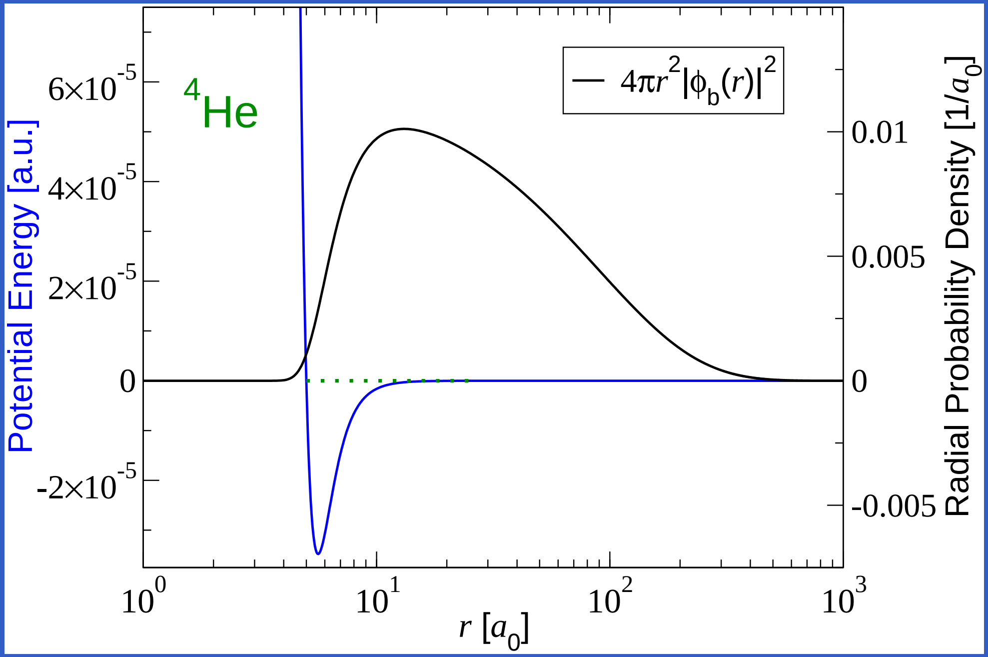

Universal bound states

Universality: Physical properties are determined by a single length scale, the s-wave scattering length, a!

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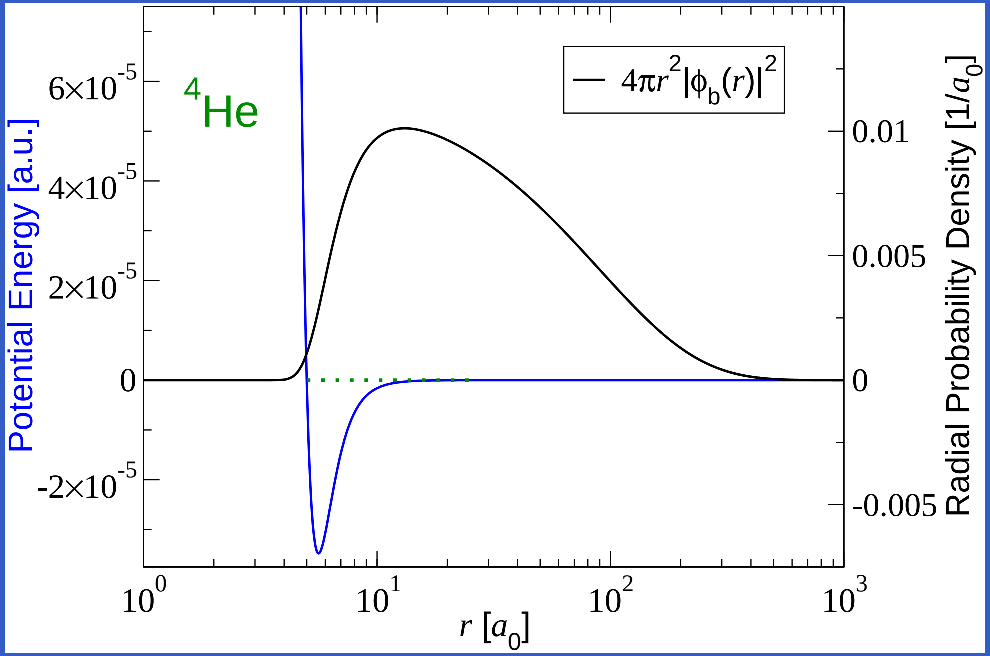
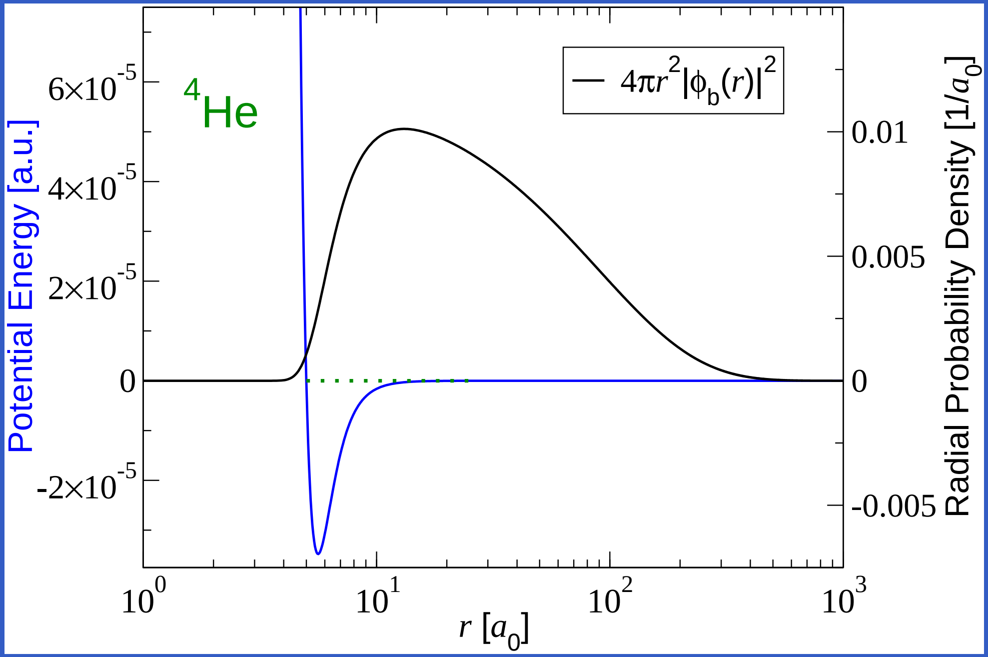


Universal bound states

Characteristic length scales

- Bond length:

$$\langle r \rangle = a/2$$



Universal bound states

Characteristic length scales

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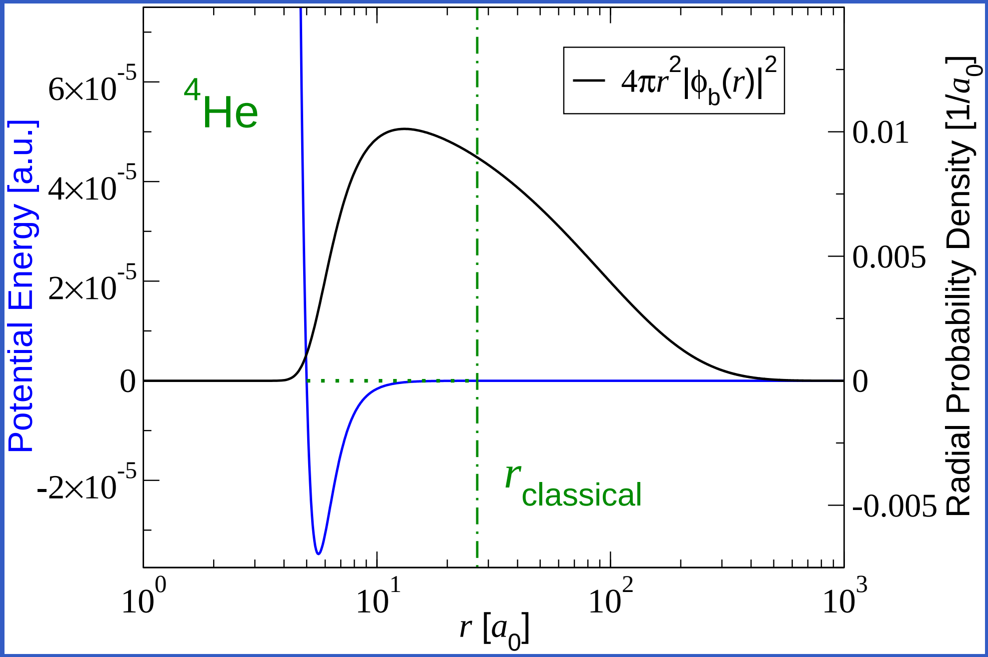
- Classical turning point:

$$r_{\text{classical}} = [a(2l_{\text{vdW}})^2]^{1/3}$$



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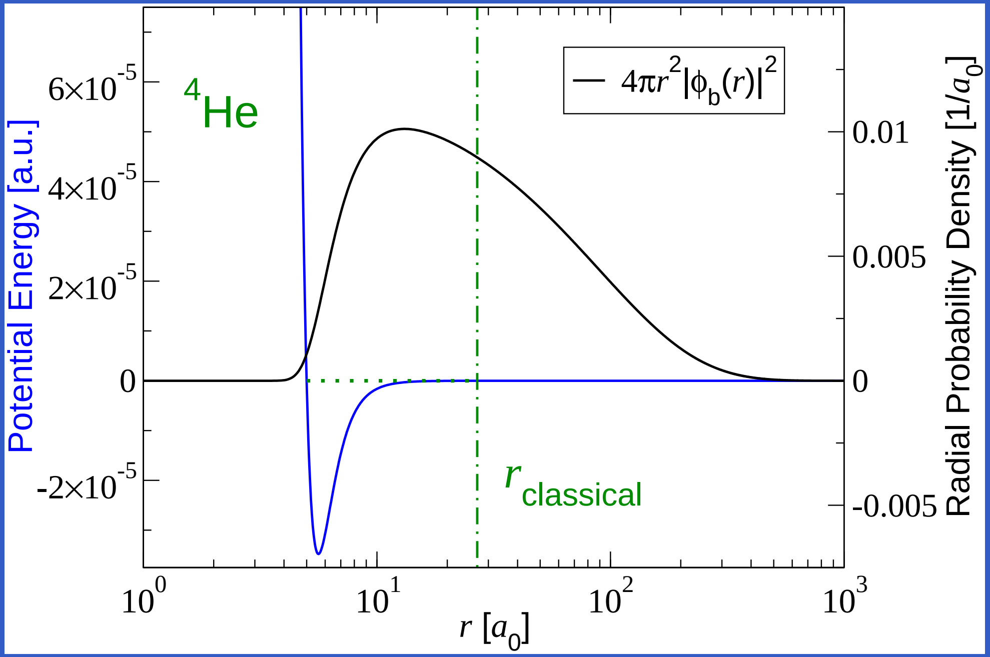

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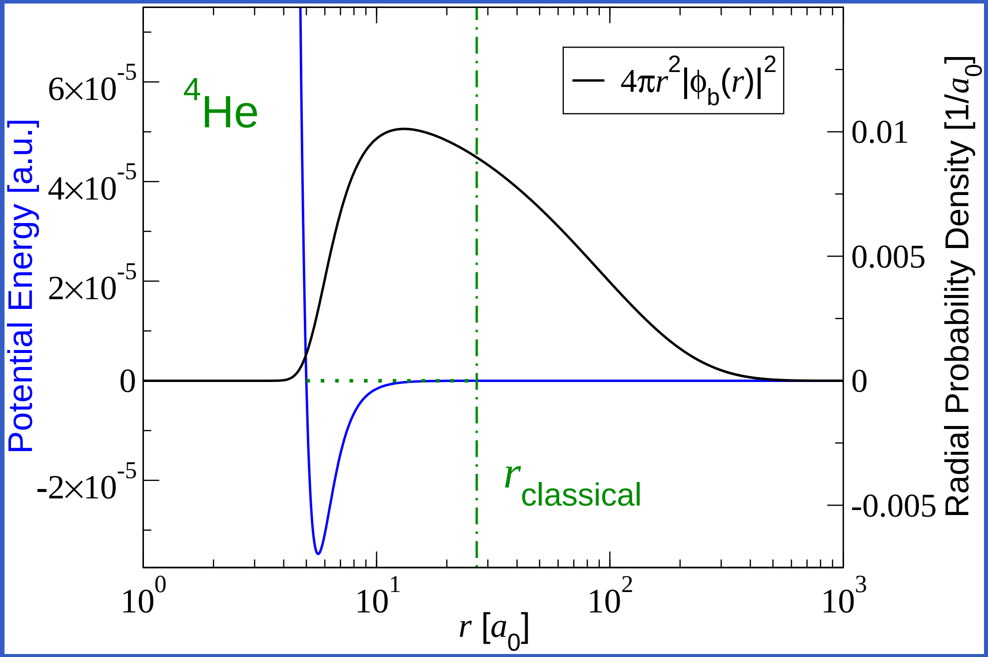
$$a \gg r_{\mathrm{classical}}$$



Universal bound states

Properties of the helium dimer

- Bond length:

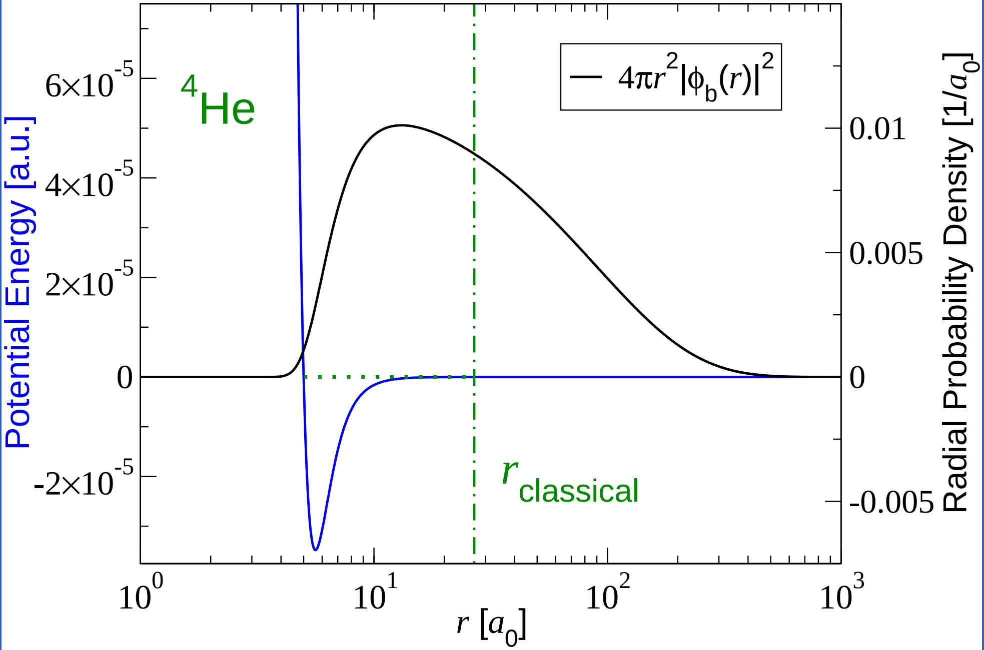
$$\langle r \rangle = 5.2 \pm 0.4\,\text{nm}$$

- Classical turning point:

$$r_{\text{classical}} = 1.4\,\text{nm}$$

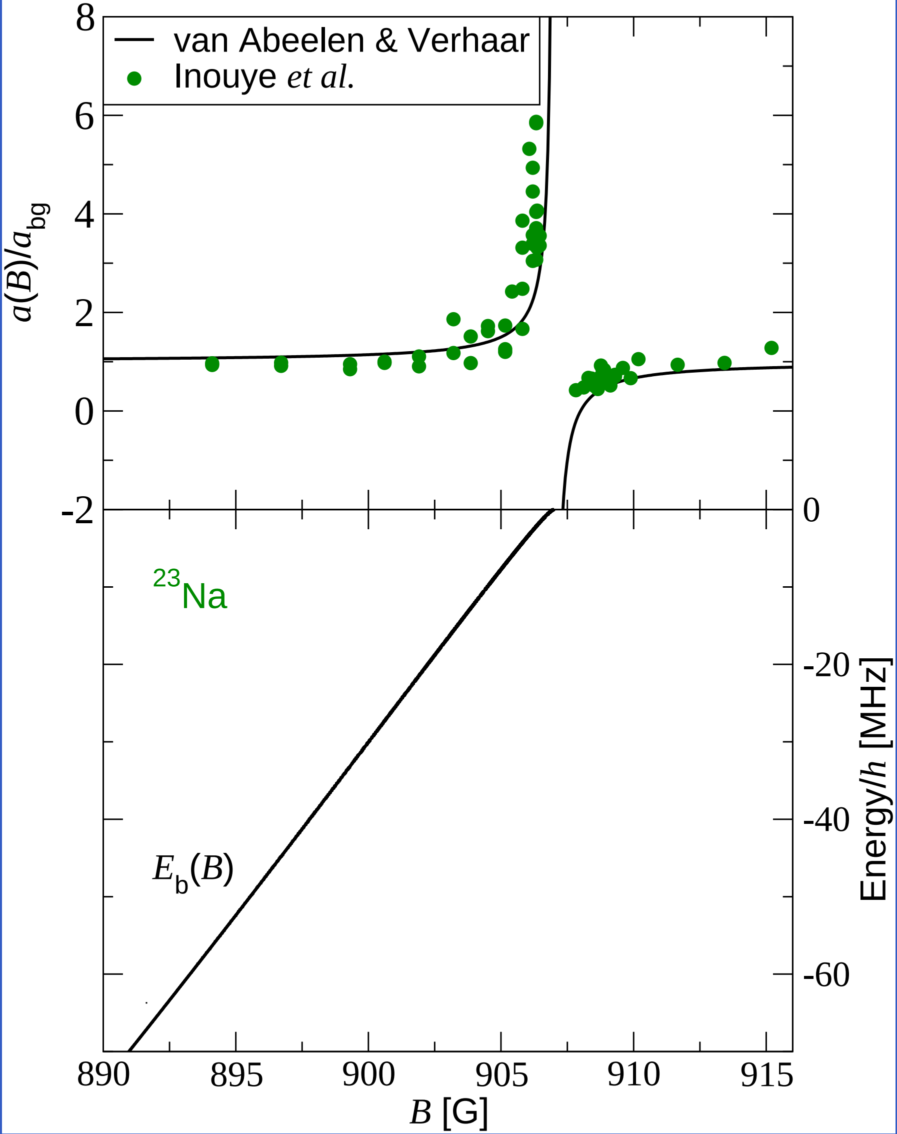
- Characteristic property:

$$a = 10.4\,\text{nm} \gg r_{\text{classical}}$$



R.E. Grisenti, W. Schöllkopf, J.P. Toennies, G.C. Hegerfeldt, TK, and M. Stoll, PRL **85**, 2284 (2000)

Magnetically-tunable interactions



The scattering length diverges.

$\Updownarrow$

A new vibrational bound state with energy $E_{\mathrm{b}}(B)$ emerges at the collision threshold.

Theory: E. Tiesinga, B.J. Verhaar, and H.T.C. Stoof, PRA **47**, 4114 (1993)
Experiment: S. Inouyé *et al.*, Nature **392**, 151 (1998)

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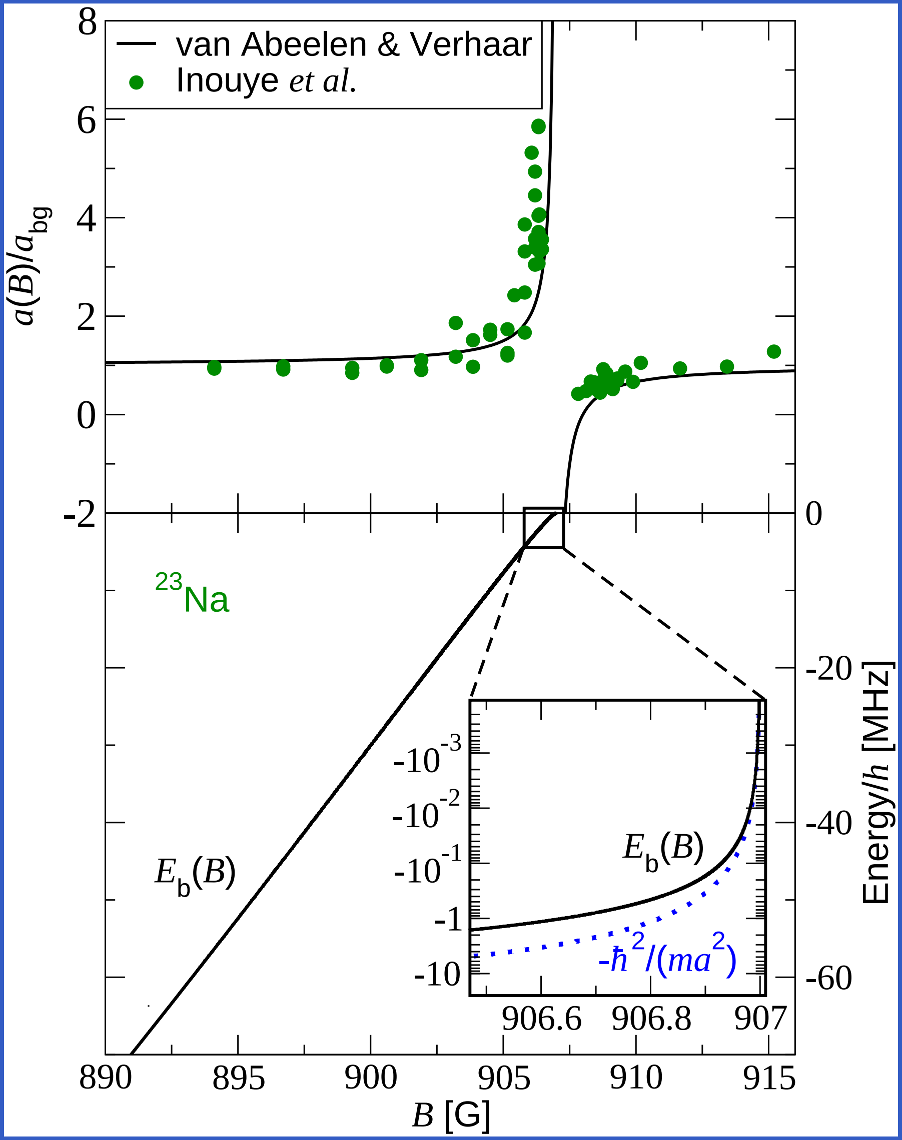

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The energy of the Feshbach molecule, $E_{\mathrm{b}}(B)$, becomes universal.

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Experiment: S. Inouyé *et al.*, Nature **392**, 151 (1998)
Review: TK, K. Góral, and P.S. Julienne, RMP **78**, 1311 (2006)

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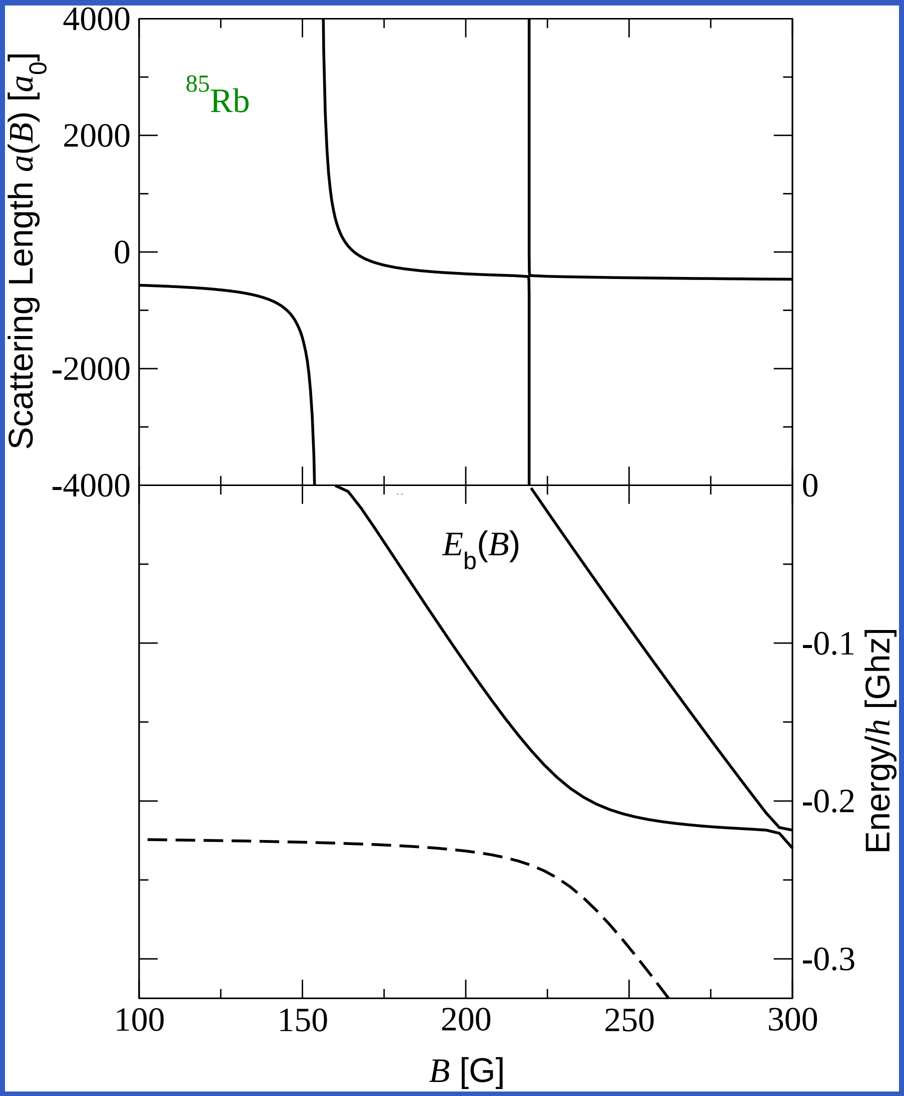

- The slope of $E_{\mathrm{b}}(B)$ with respect to the magnetic-field strength, B, varies between different atomic species.

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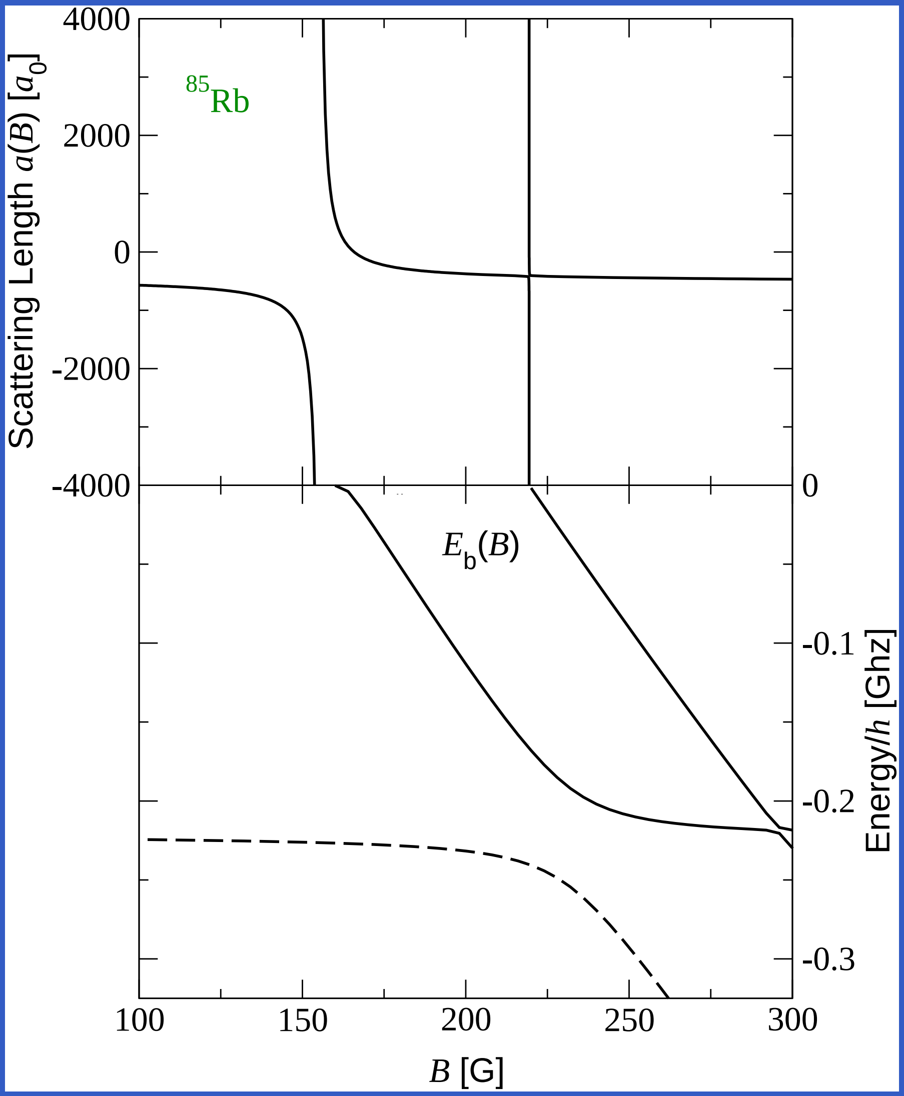

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- Background-scattering lengths away from singularities can differ between atomic species.

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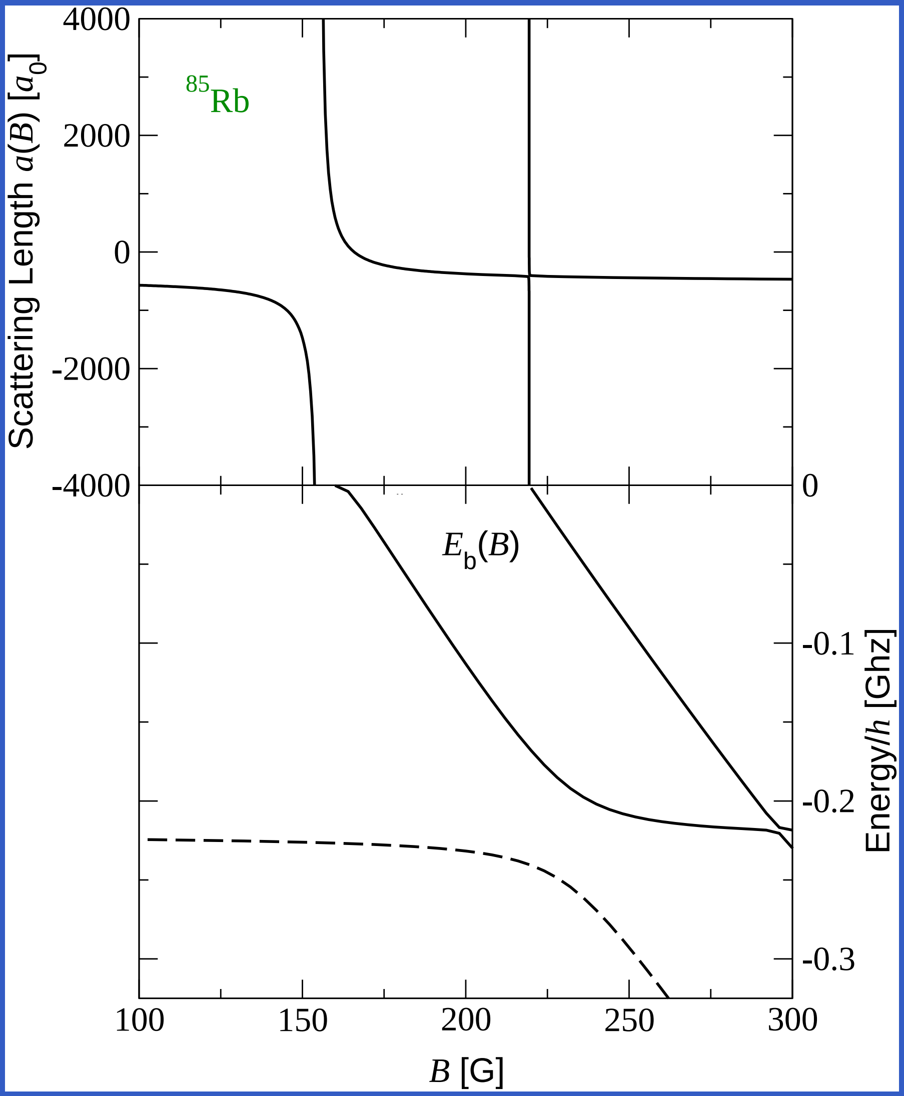

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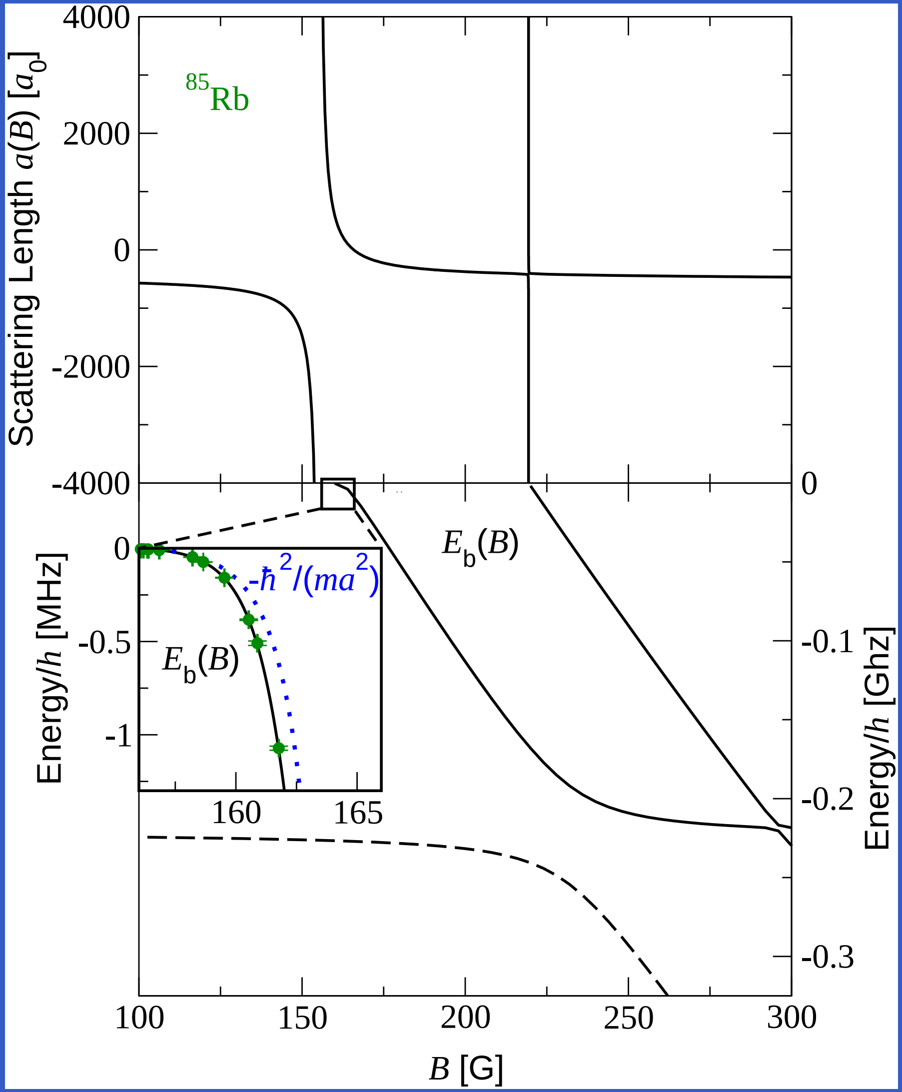

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- Background-scattering lengths away from singularities can differ between atomic species.
- Singularities of the scattering length can have different widths.
- The region of universality can differ between Feshbach resonances.

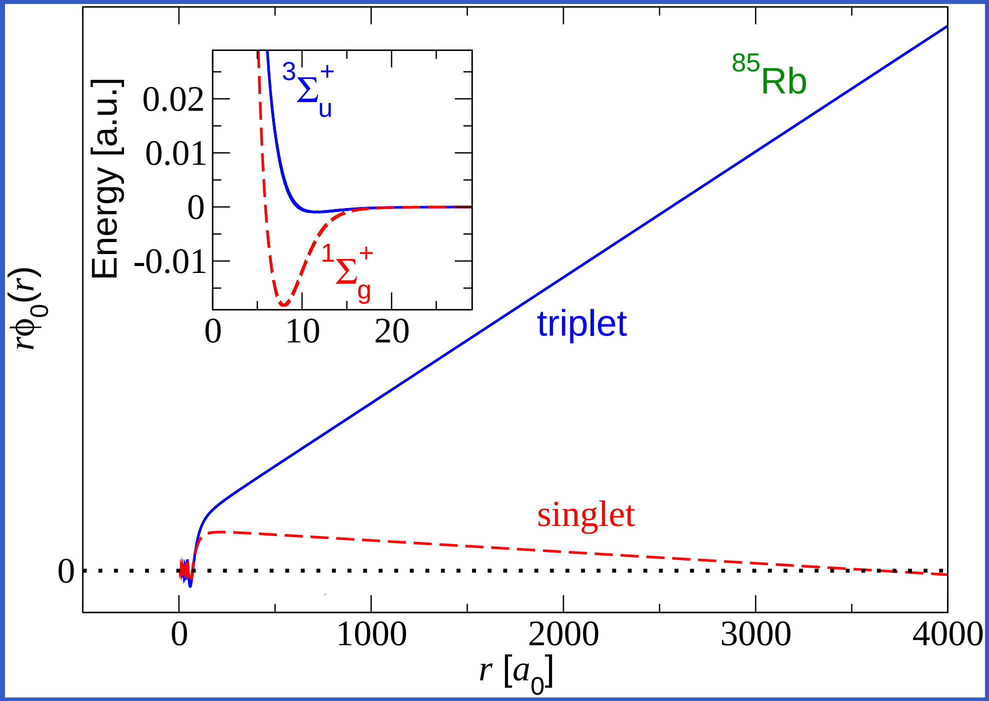
N.R. Claussen *et al.*, PRA **67**, 060701(R) (2003)

Magnetically-tunable interactions

How does one determine a scattering length?

- Radial Schrödinger equation:

$$\left[-\frac{\hbar^2}{m}\frac{d^2}{dr^2}+V(r)\right]r\phi_0(r)=0$$



J.R. Taylor, *Scattering Theory* (Wiley, New York, 1972)

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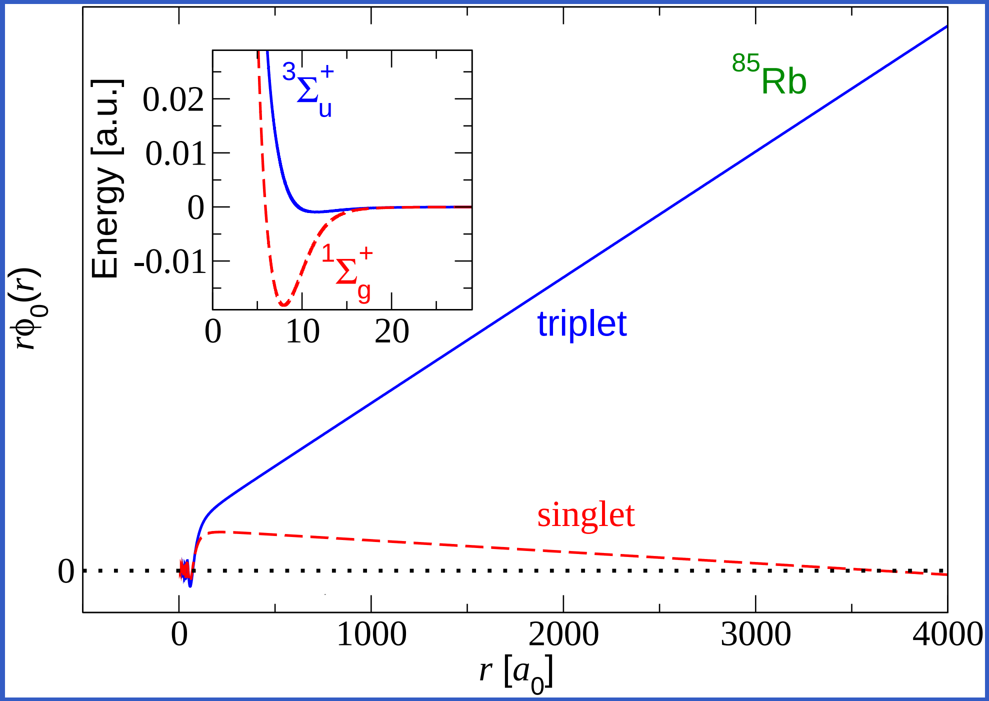

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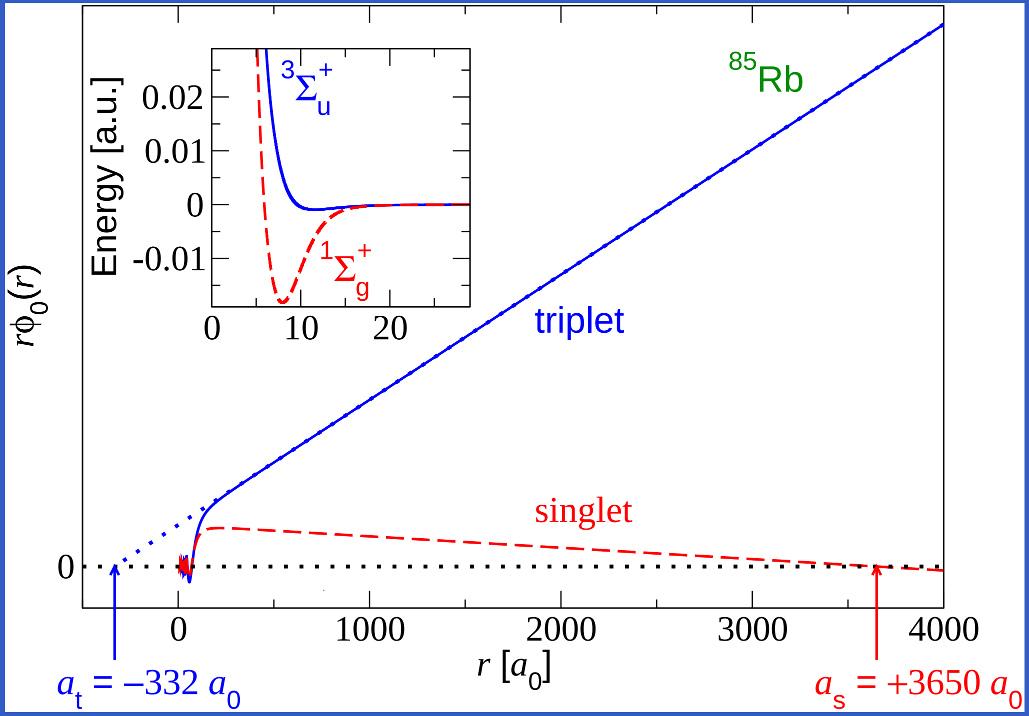
- Asymptotic behaviour as $r \to \infty$:

$$r\phi_0(r) \propto r - a$$



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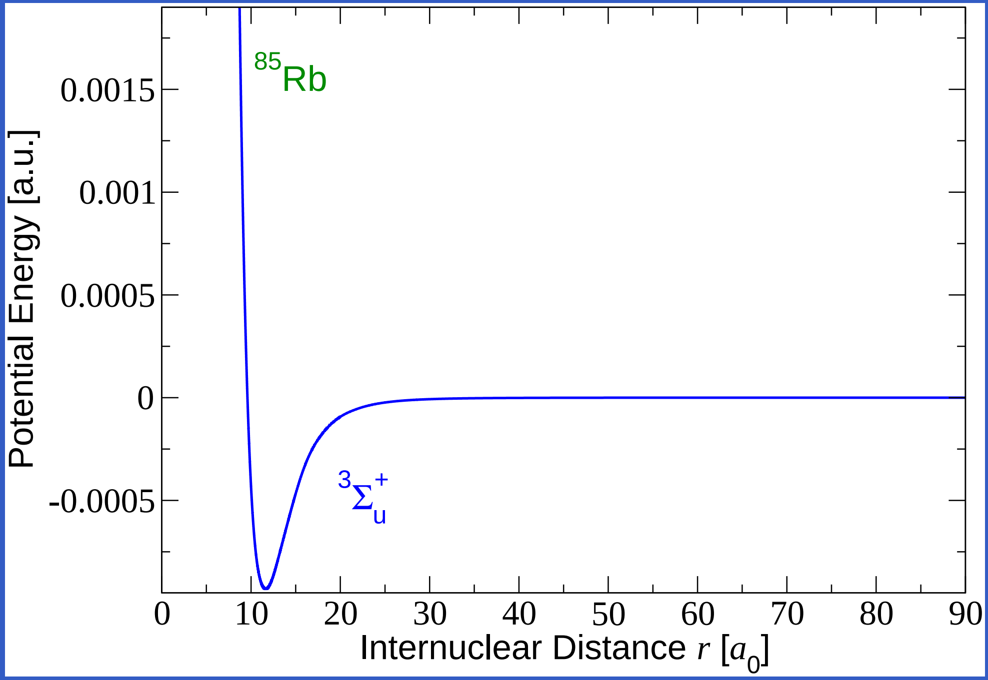
Scattering length of an alkali potential

- Gibakin-Flambaum formula:

$$a = \bar{a}\left[1 - \tan(\varphi_{\mathrm{WKB}} - \pi/8)\right]$$



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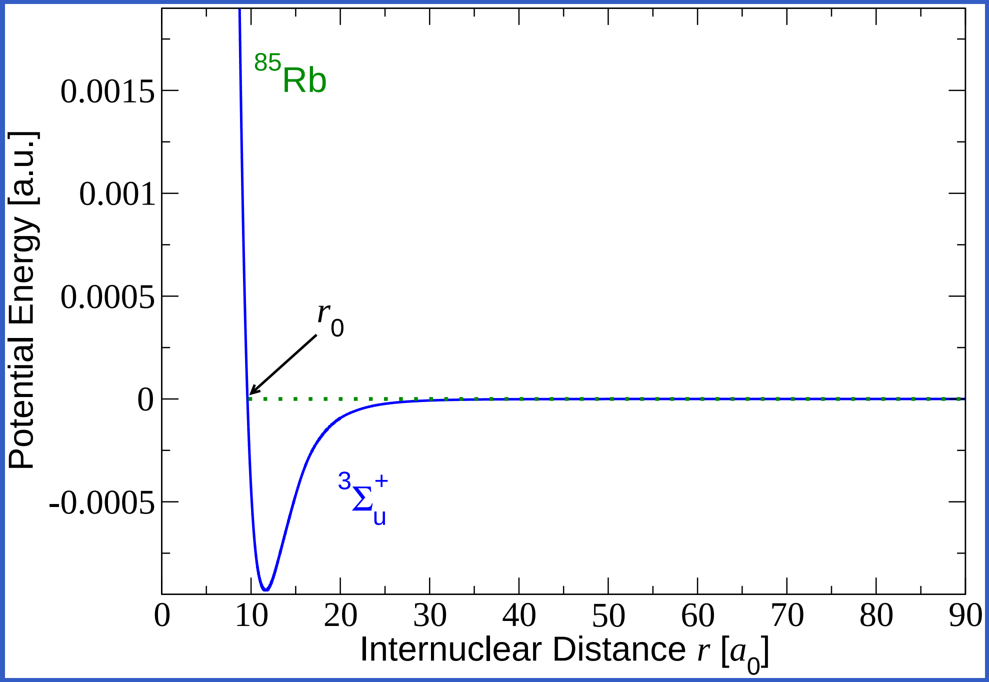

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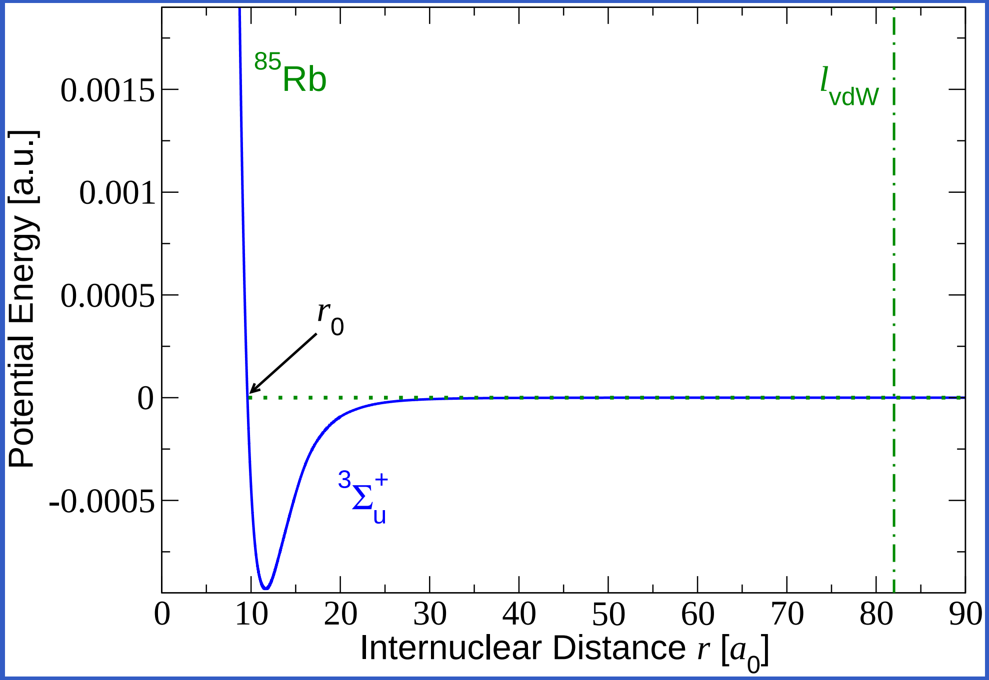
- Mean scattering length:

$$\bar{a} = 0.95598\, l_{\mathrm{vdW}}$$



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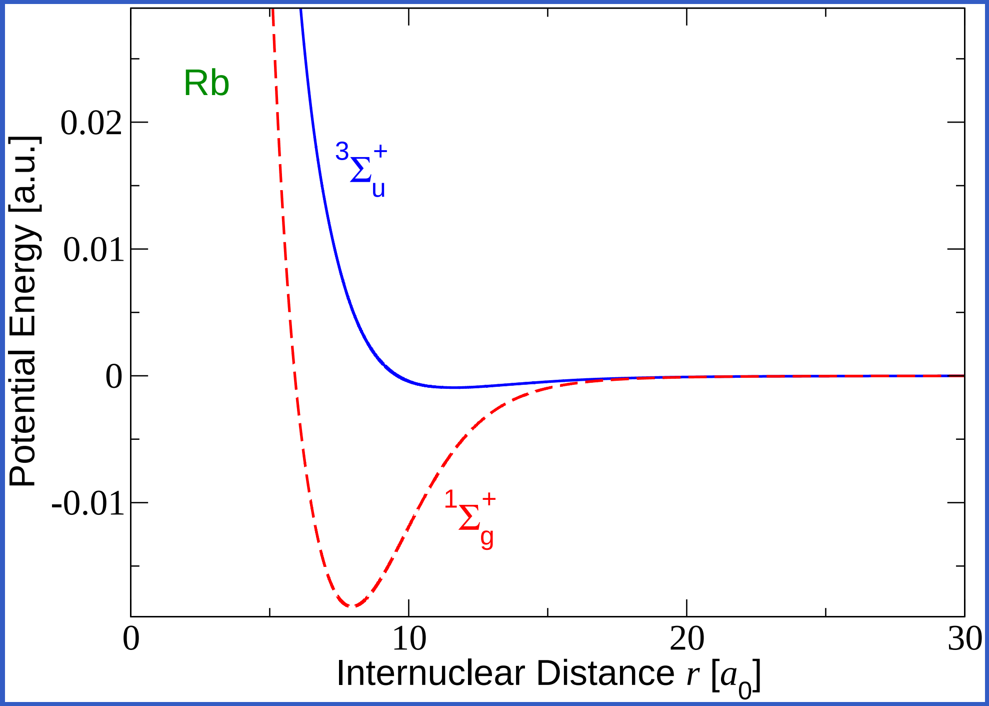
What is coupled-channels theory?

- Singlet and triplet potentials describe interactions for given total electronic spin quantum numbers of an alkali-atom pair, $S = 0, 1$, respectively.



General theory: H.T.C. Stoof, J.M.V.A. Koelman, and B.J. Verhaar, PRB **38**, 4688 (1988)

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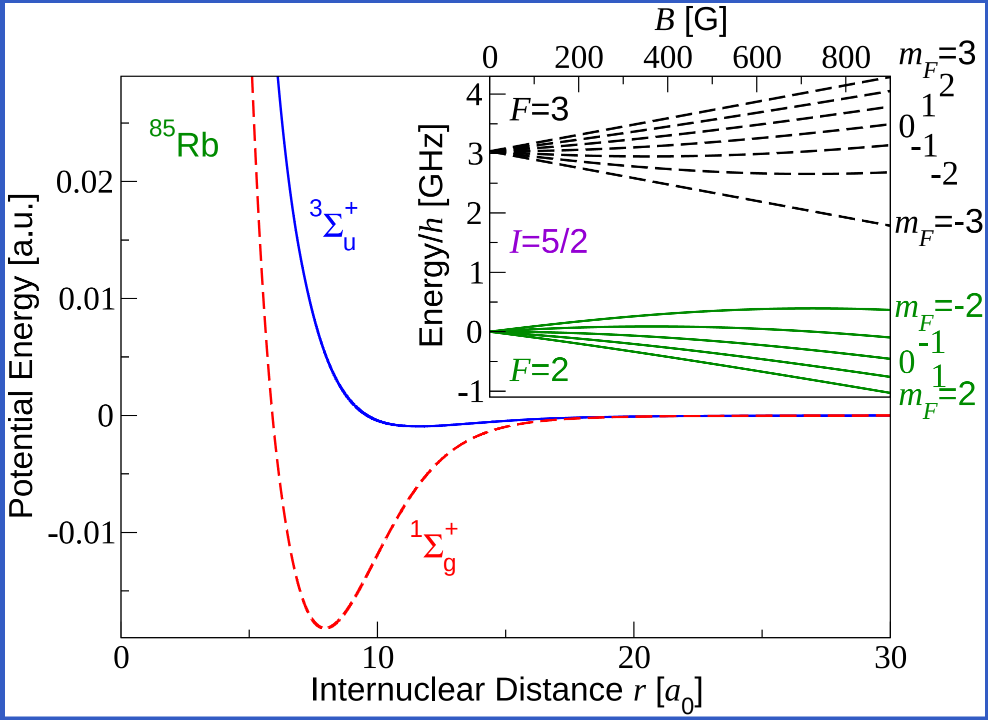
- Asymptotic scattering channels are described by pairs of Zeeman states in terms of angular momentum quantum numbers of each atom.



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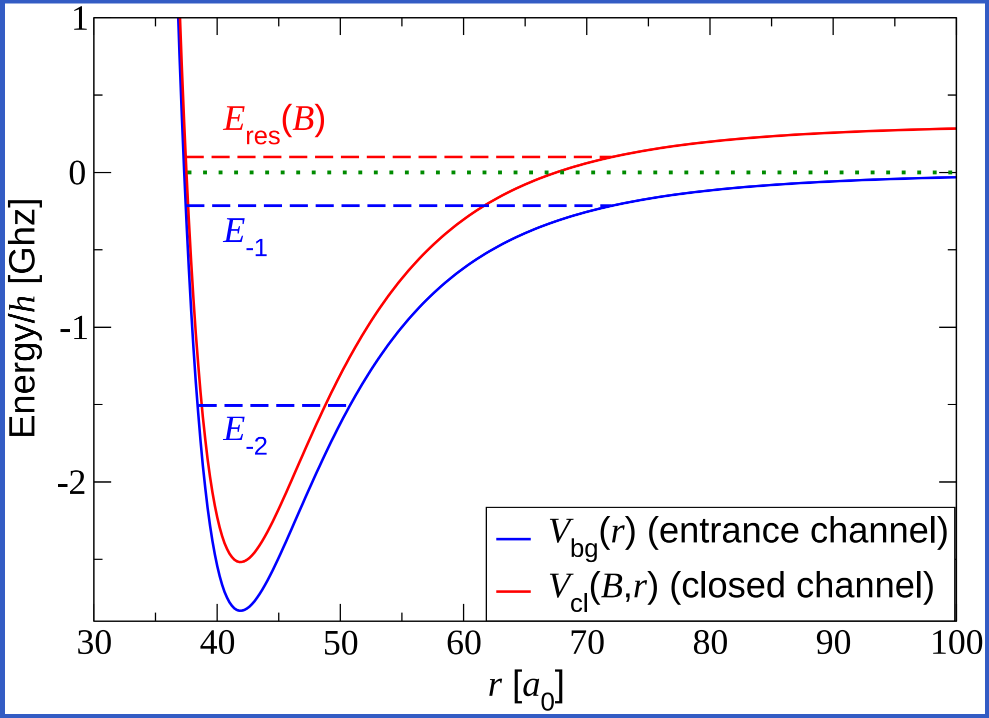
Hyperfine structure in alkali atoms: E. Arimondo, M. Inguscio, and P. Violino, RMP **49**, 31 (1977)

Magnetically-tunable interactions

Feshbach resonances

- Two-channel Hamiltonian:

$$H_{2B} = \begin{pmatrix} H_{bg} & \\ & H_{cl}(B) \end{pmatrix}$$



A.J. Moerdijk, B.J. Verhaar, and A. Axelsson, PRA **51**, 4852 (1995)
F.H. Mies, E. Tiesinga, and P.S. Julienne, PRA **61**, 022721 (2000)

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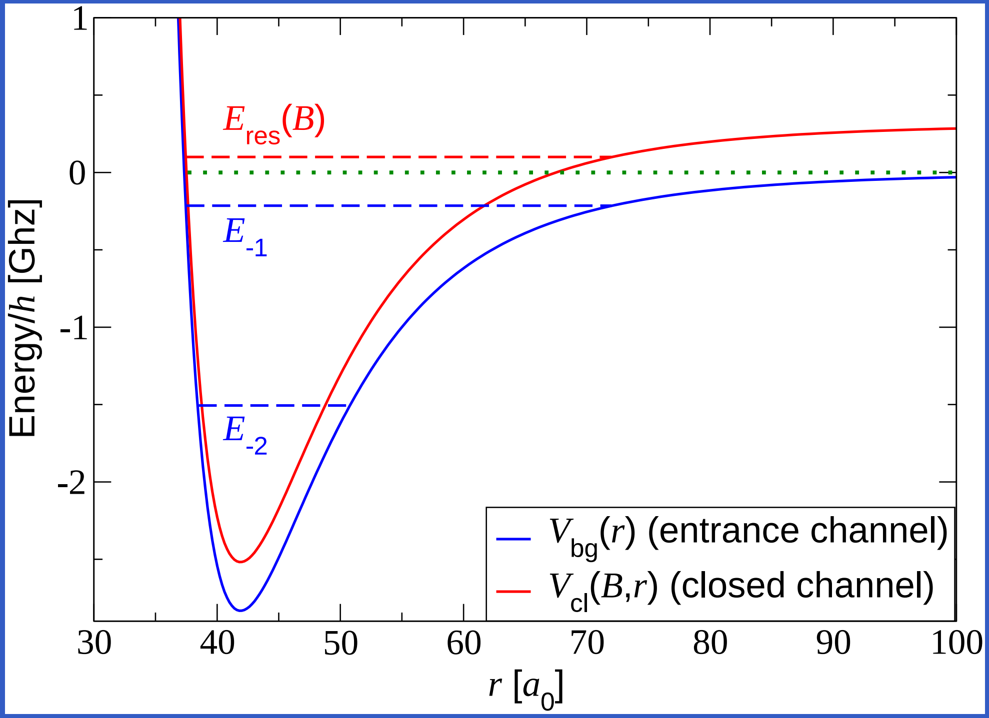

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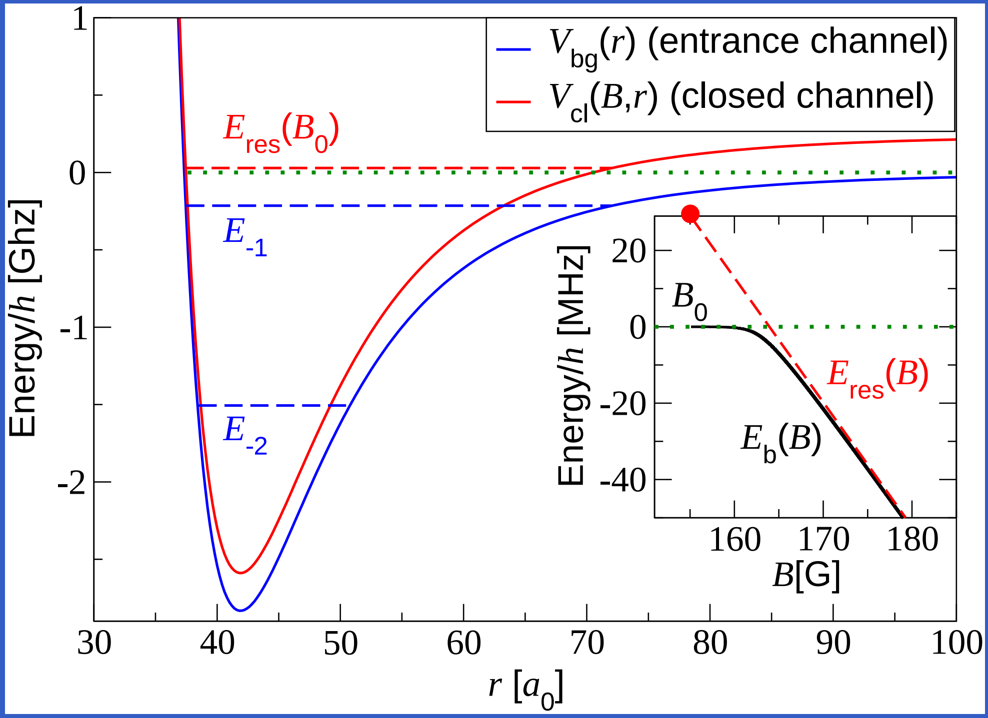

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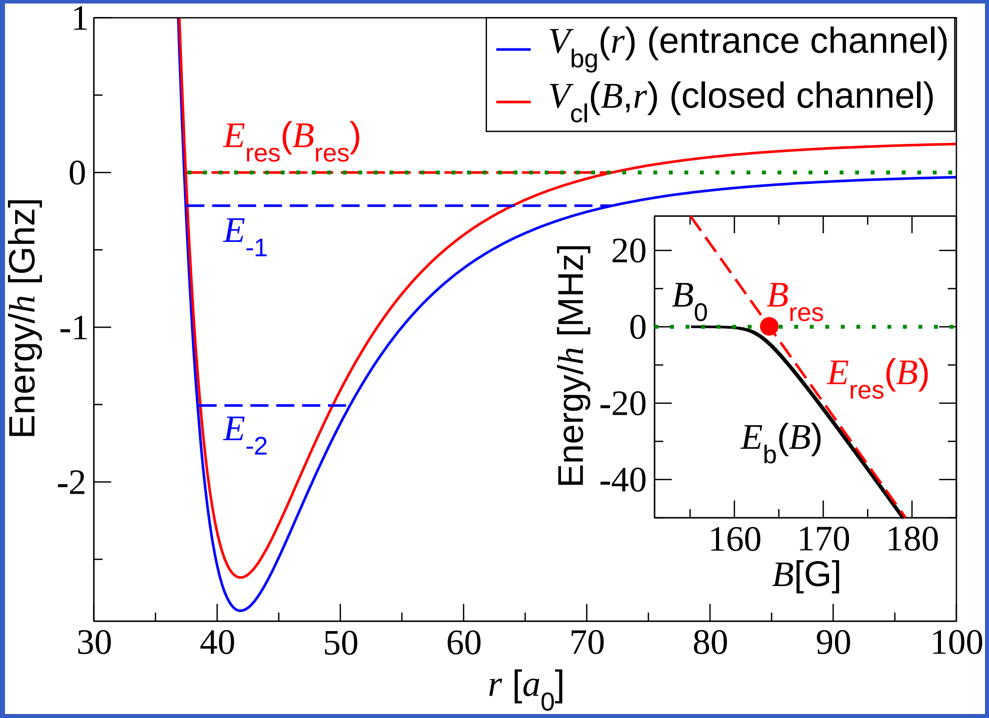

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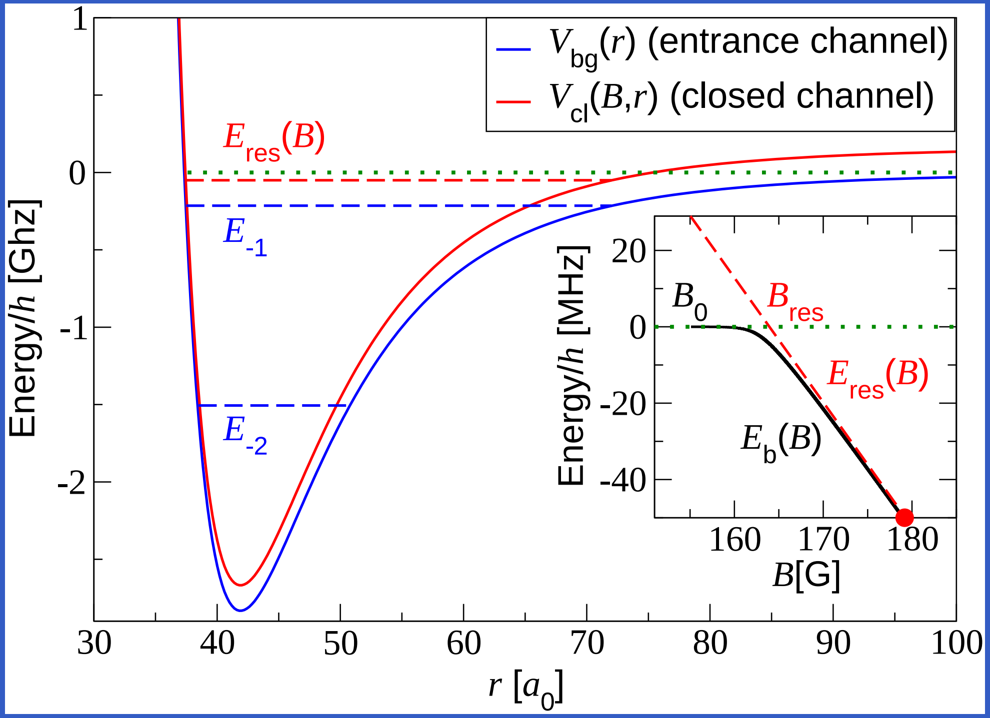

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Two-channel versus coupled-channels theory

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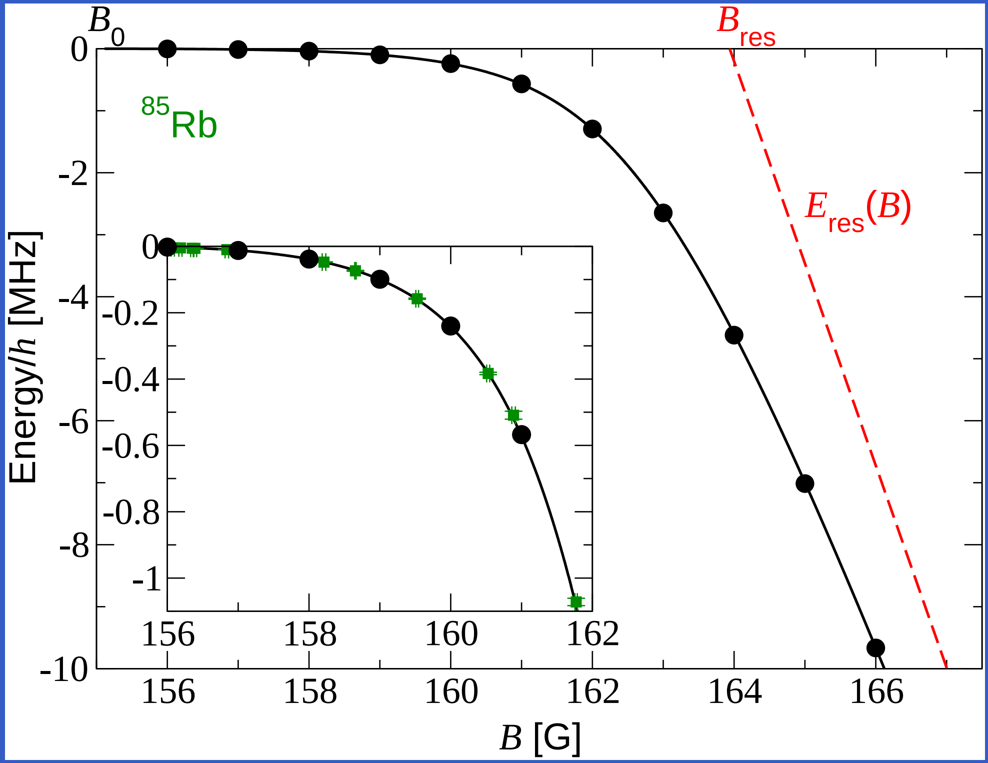


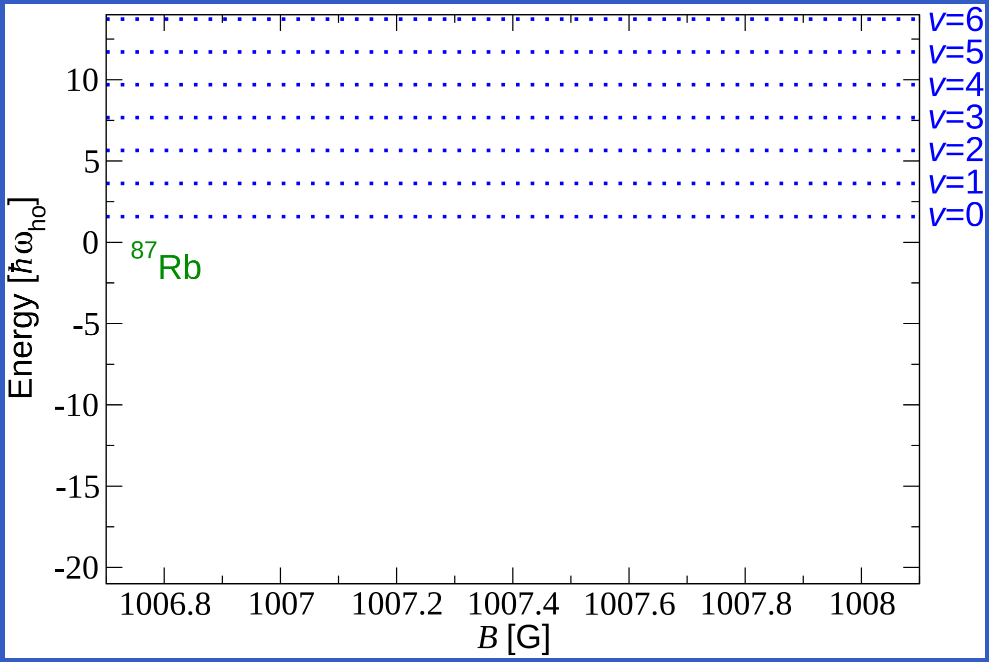
Coupled-channels theory and experiment: N.R. Claussen, S.J.J.M.F. Kokkelmans, S.T. Thompson, E.A. Donley, E. Hodby, and C.E. Wieman, PRA **67**, 060701(R) (2003)
This two-channel approach: K. Góral, TK, and K. Burnett, PRA **71**, 023603 (2005)

Magnetically-tunable interactions

Feshbach-molecule association in a harmonic atom trap

- Entrance-channel Hamiltonian:

$$H_{\rm bg} = -\frac{\hbar^2}{m}\nabla^2 + V_{\rm bg}(r) + V_{\rm ho}(r)$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

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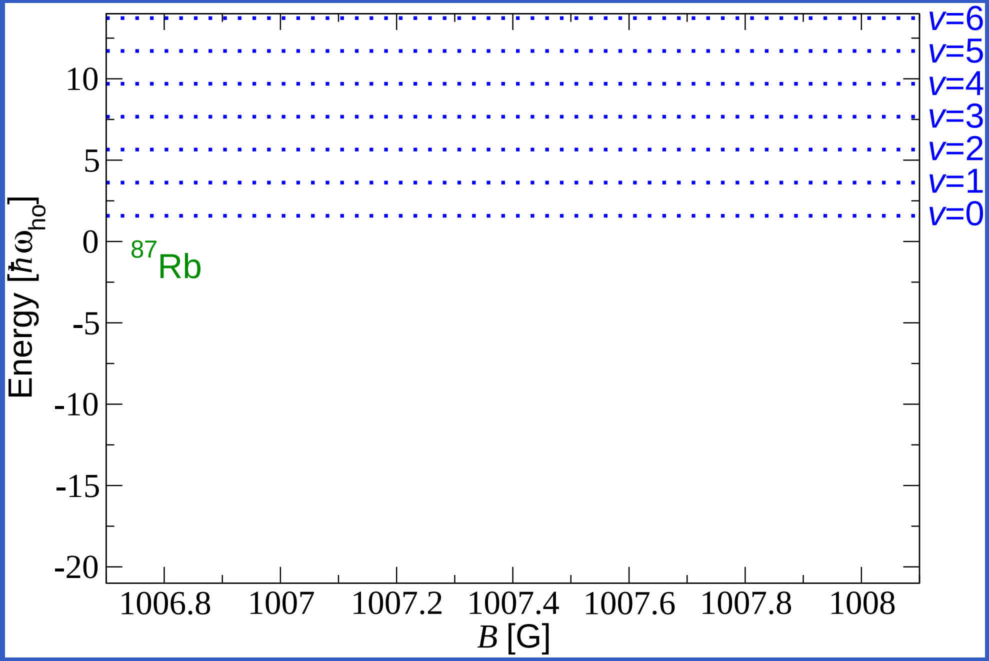

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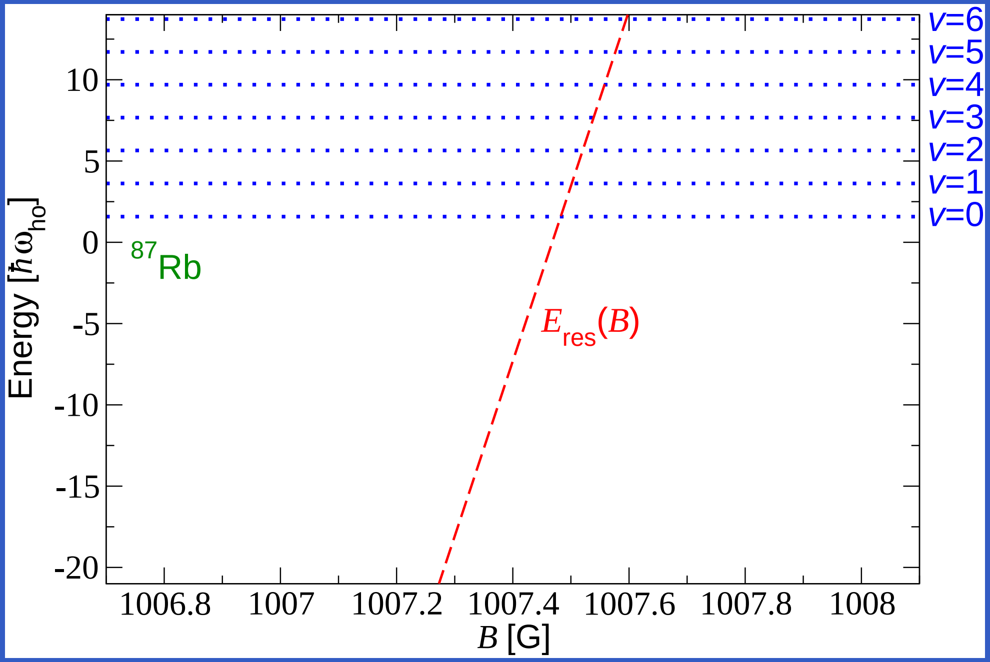

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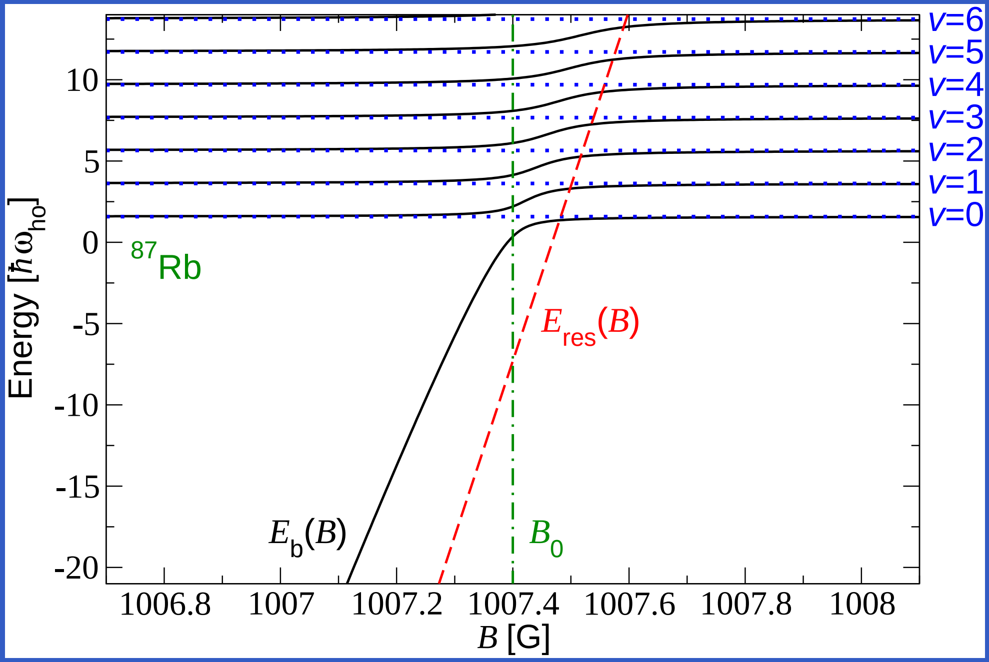

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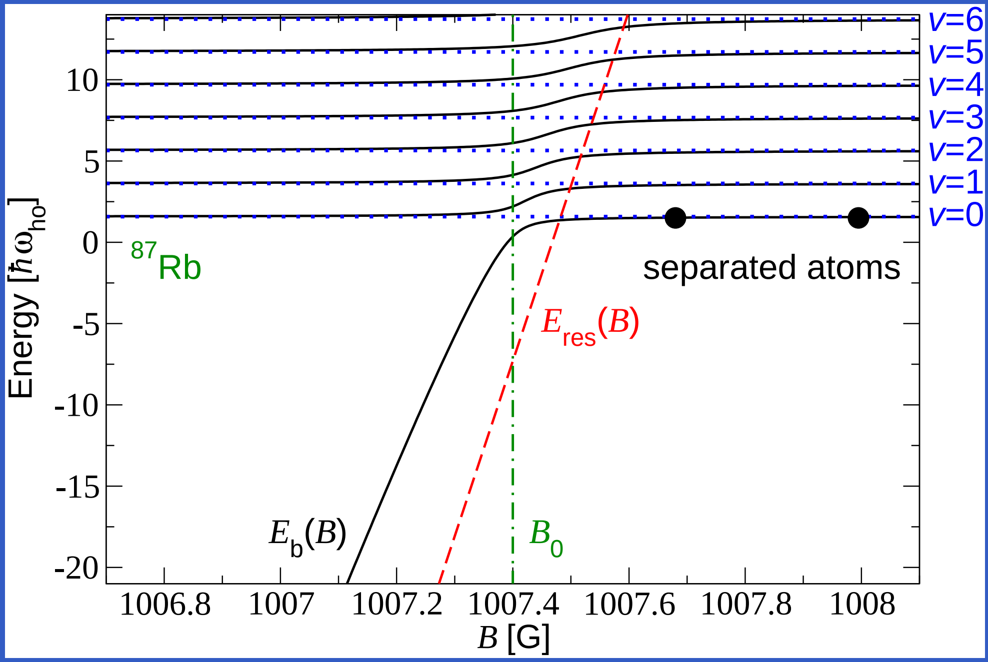

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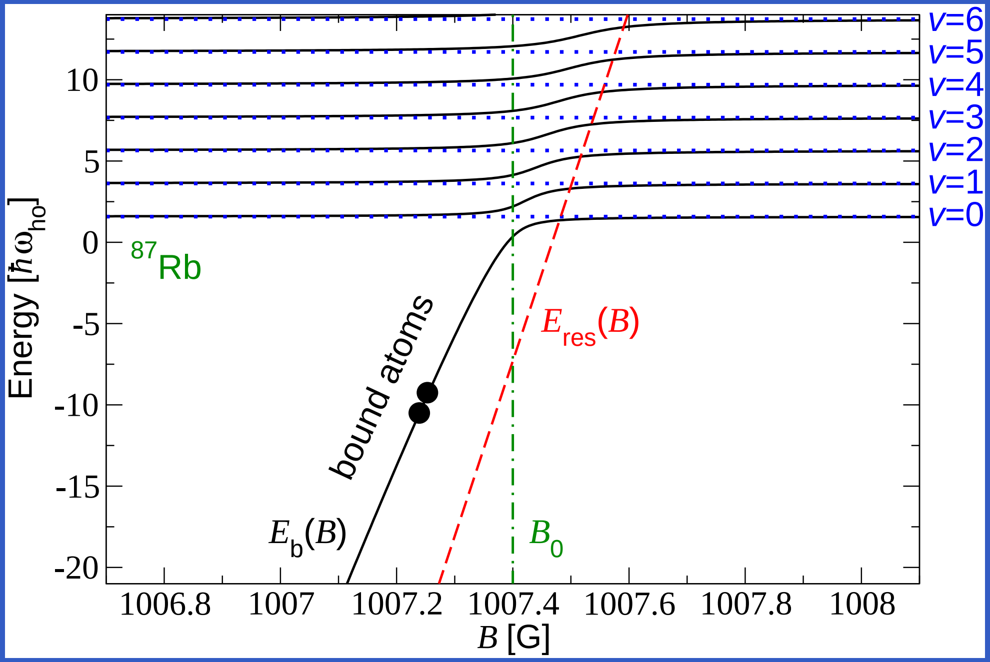

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Magnetically-tunable interactions

Resonance shape

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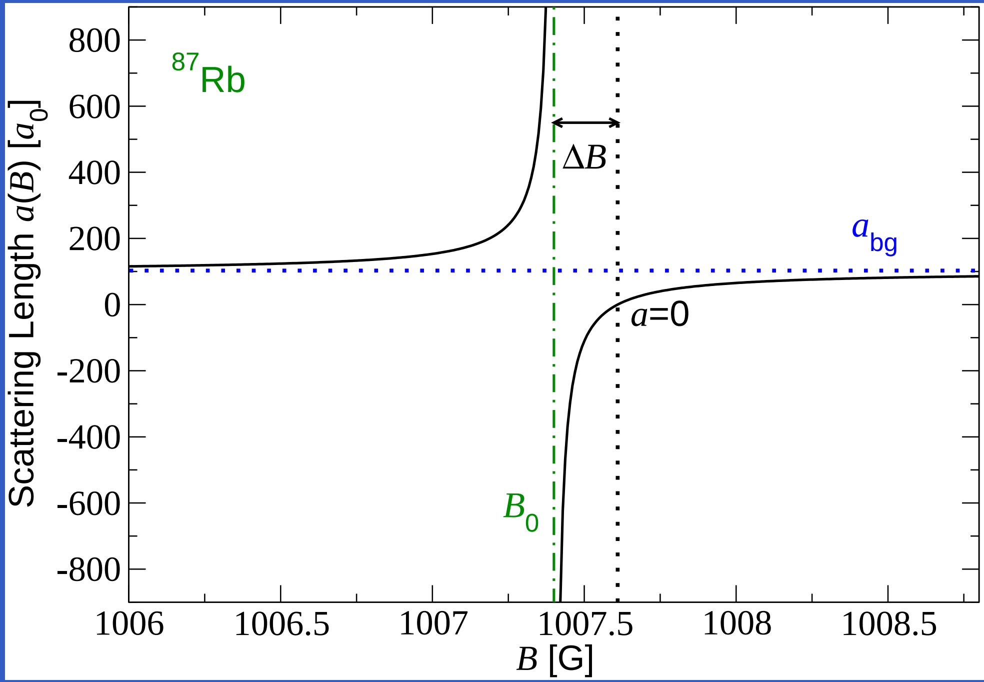
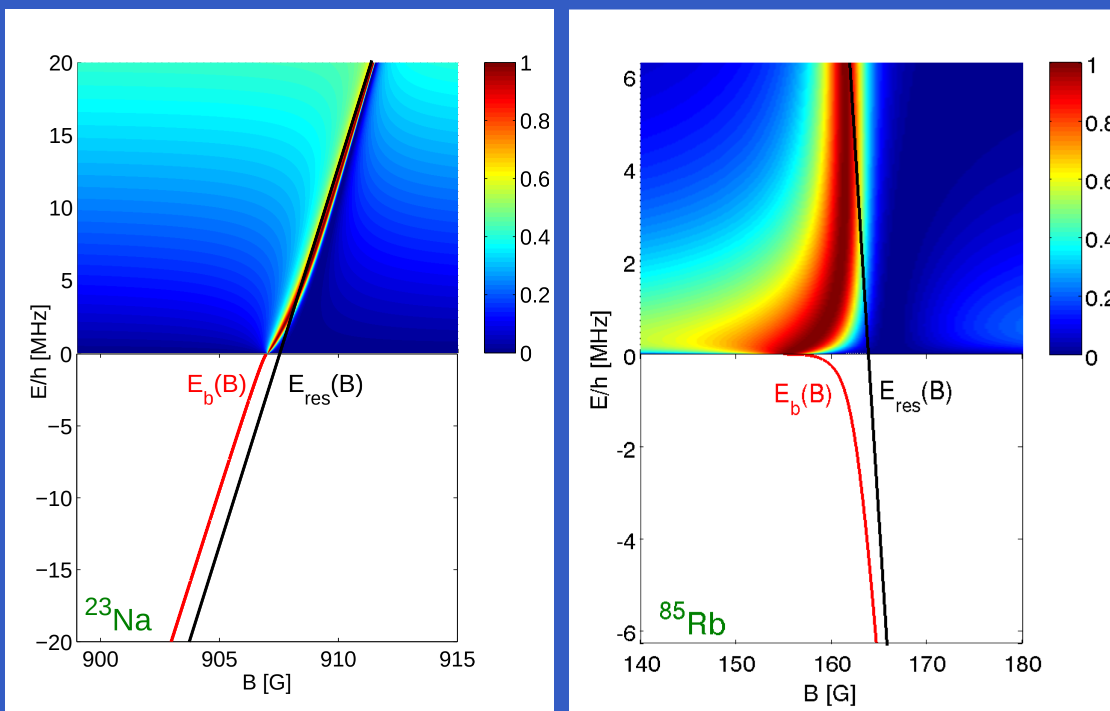


A.J. Moerdijk, B.J. Verhaar, and A. Axelsson, PRA **51**, 4852 (1995)

Classification of Feshbach resonances

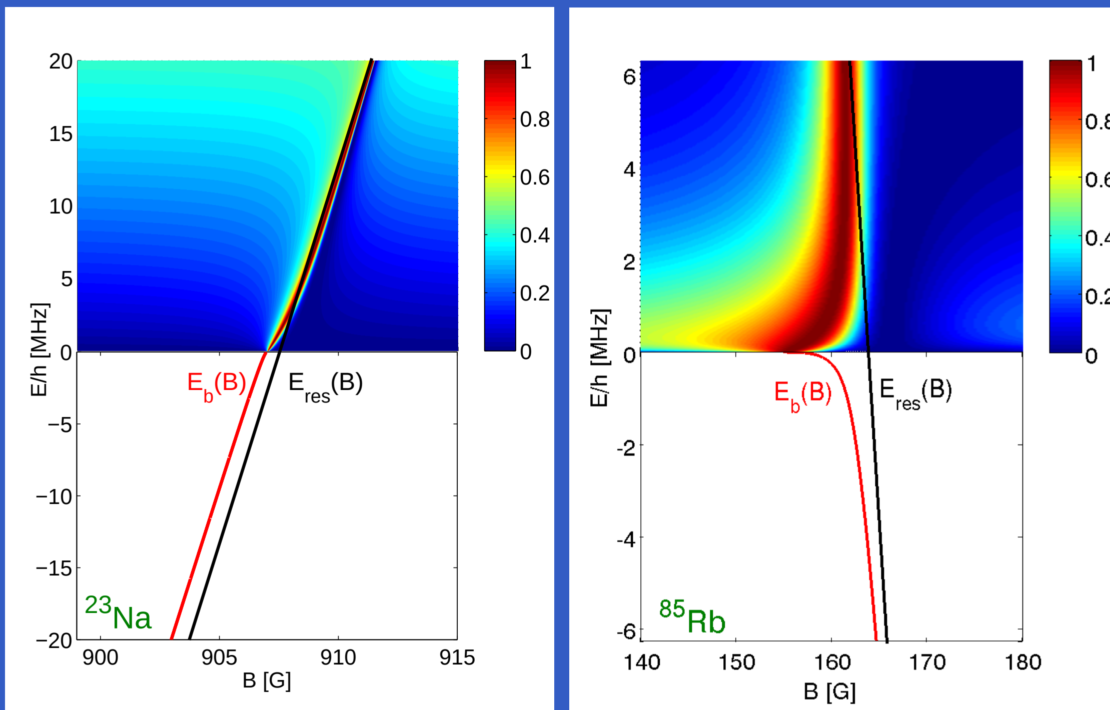


- s-wave scattering cross section:

$$\sigma_0(k) = 8\pi k^{-2} \sin^2 \delta_0(k) \underset{k\to 0}{\sim} 8\pi a^2$$

P.S. Julienne and B. Gao, *Simple theoretical models for resonant cold atom interactions*, in Atomic Physics **20**, ed. by C.Roos, H. Häffner, and R. Blatt (American Institute of Physics, Conference Proceedings **869**, 2006), pp. 261-268, e-print physics/0609013
B. Marcelis *et al.*, PRA **70**, 012701 (2004)
Figures: Tom Hanna

Classification of Feshbach resonances



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Figures: Tom Hanna

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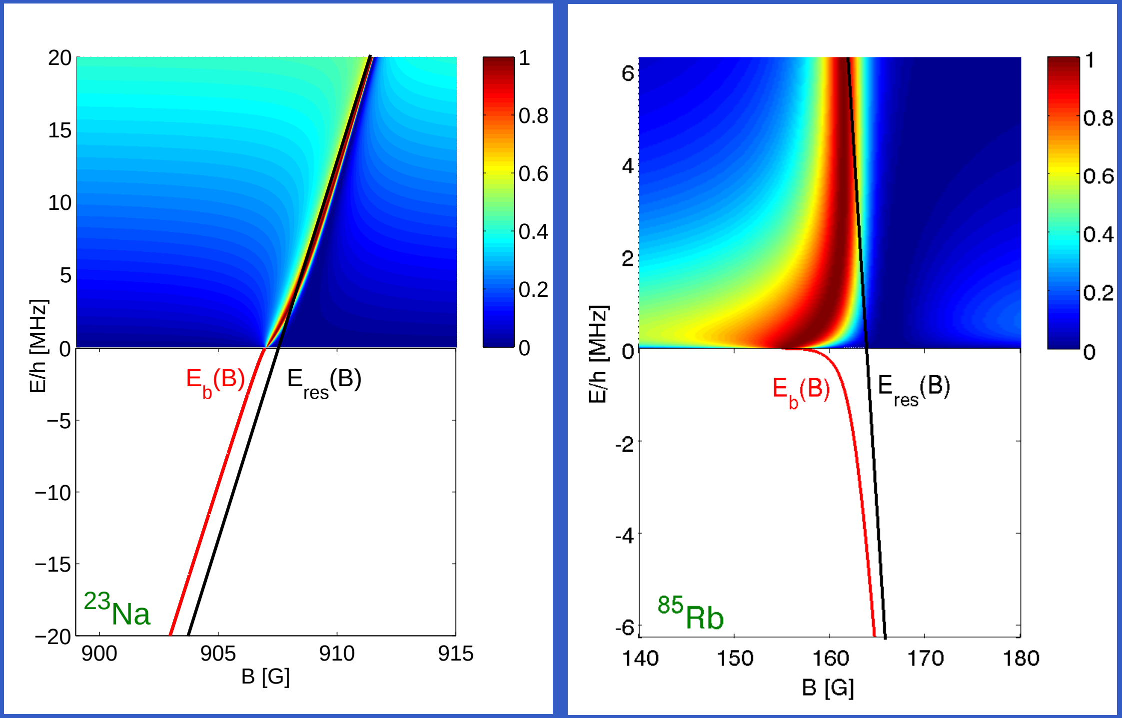

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Figures: Tom Hanna

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- Width and behaviour of the resonance energy, $E_{\mathrm{res}}(B)$, above the collision threshold differ between ^{23}Na and ^{85}Rb.

Classification of Feshbach resonances

When is a single-channel treatment appropriate?

- Single-channel Hamiltonian:

$$H_{2\mathrm{B}}^{\mathrm{eff}} = -\frac{\hbar^2}{m}\boldsymbol{\nabla}^2 + V_{\mathrm{eff}}(B, r)$$

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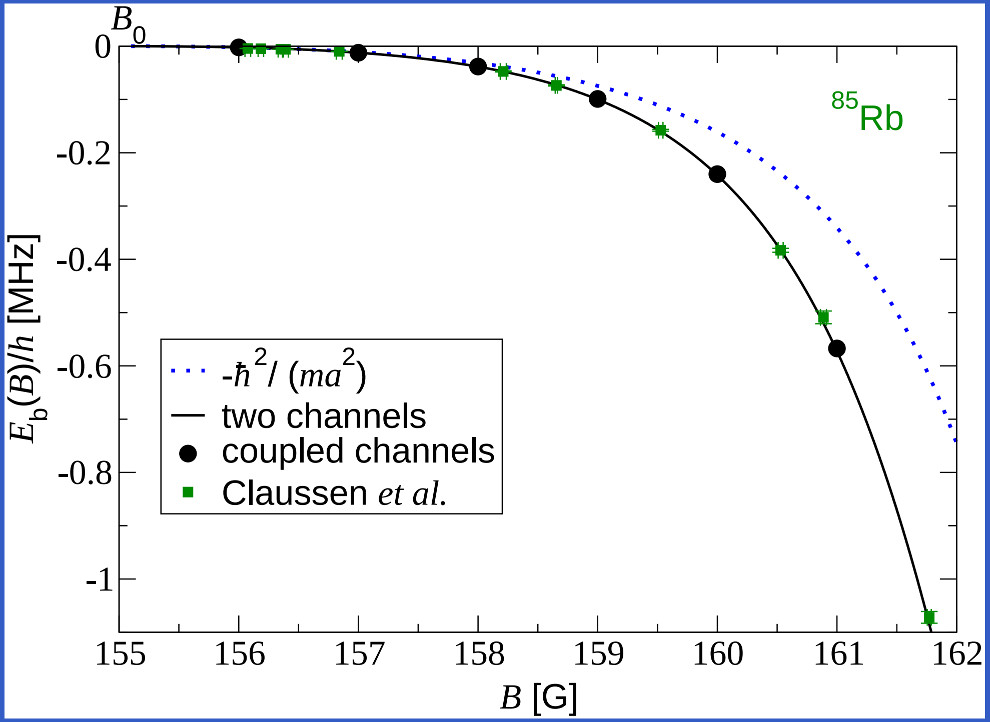

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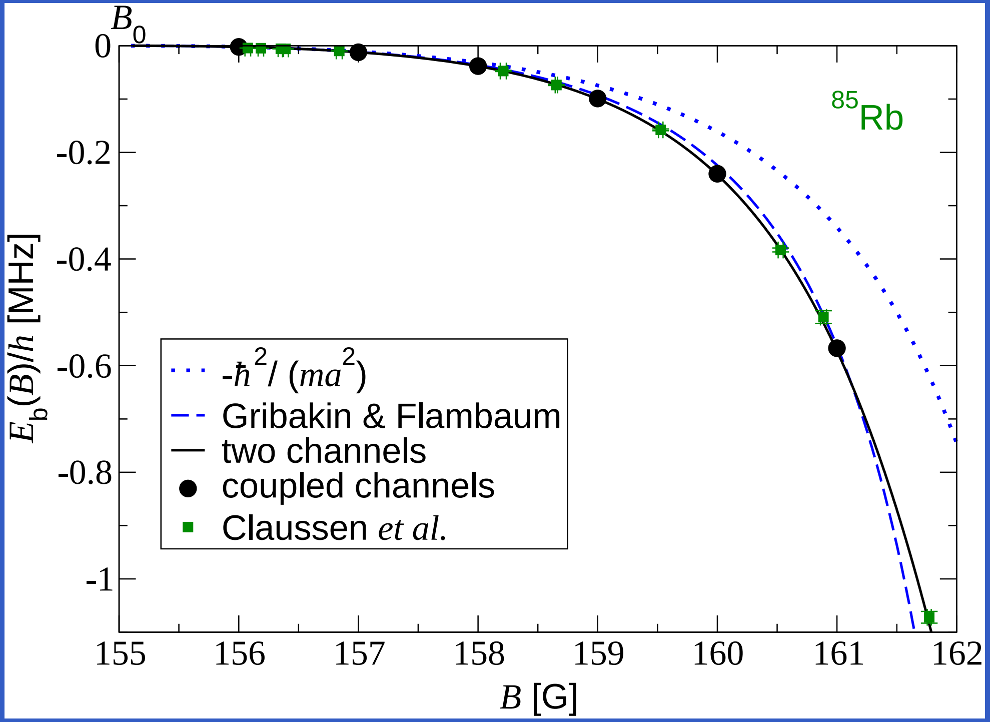
$$E_{\text{b}} = -\hbar^2/[m(a-\bar{a})^2]$$



G.F. Gribakin and V.V. Flambaum, PRA **48**, 546 (1993)
B. Gao, J. Phys. B **37**, 4273 (2004)

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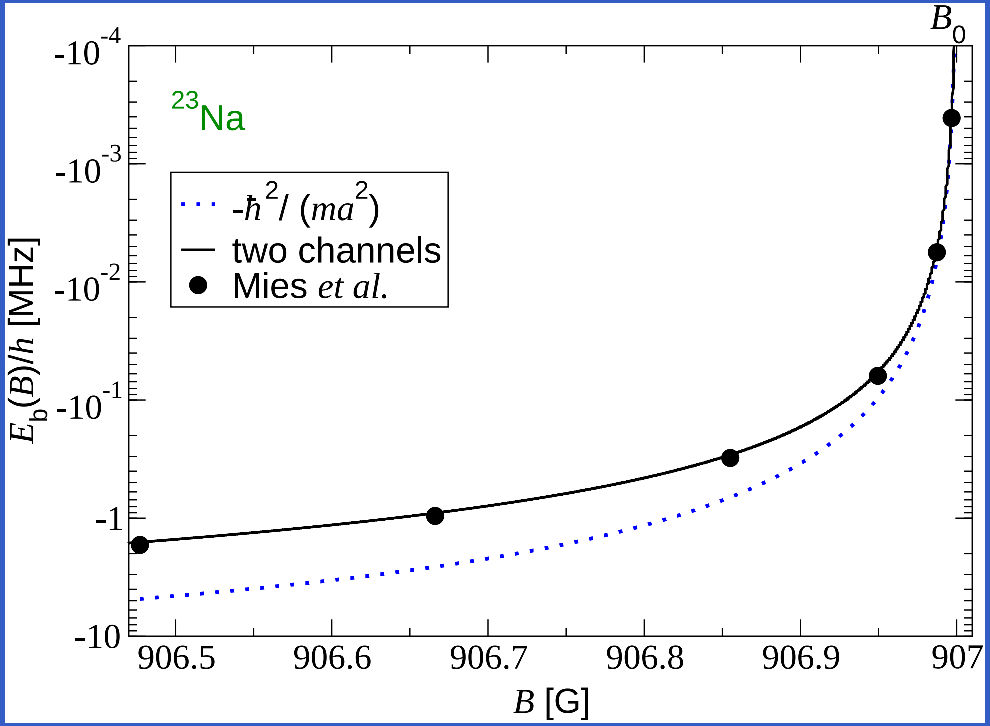

This two-channel two-potential model: F.H. Mies, E. Tiesinga, and P.S. Julienne, PRA **61**, 022721 (2000)
This two-channel single-resonance approach: TK, K. Góral, and T. Gasenzer, PRA **70**, 023613 (2004)

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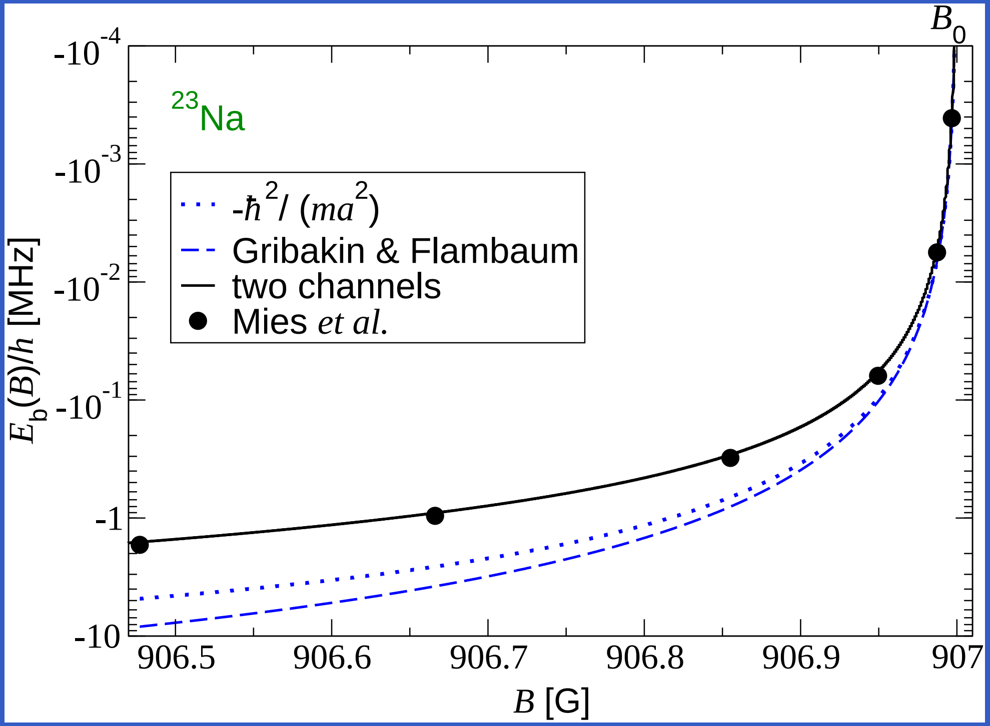

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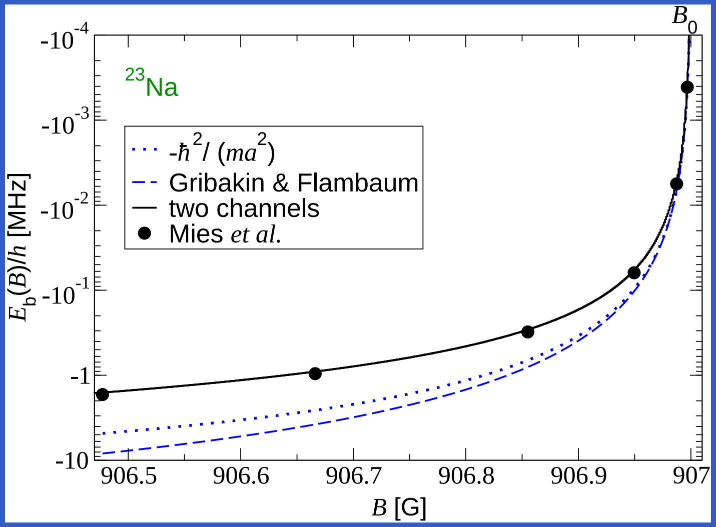
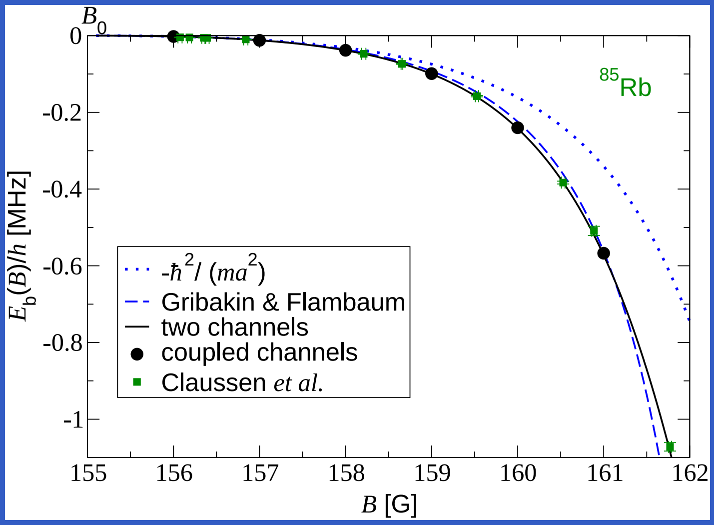


closed-channel dominated



entrance-channel dominated

$$\left| E_\mathrm{b}(B) + \frac{\hbar^2}{m[a(B)-\bar{a}]^2} \right| < \left| E_\mathrm{b}(B) + \frac{\hbar^2}{ma^2(B)} \right|$$

TK, K. Góral, and T. Gasenzer, PRA **70**, 023613 (2004)
See also: D.S. Petrov, PRL **93**, 143201 (2004)

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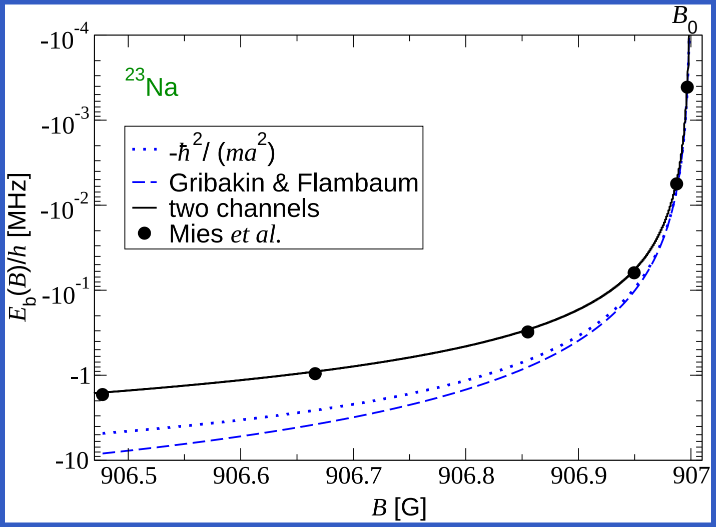

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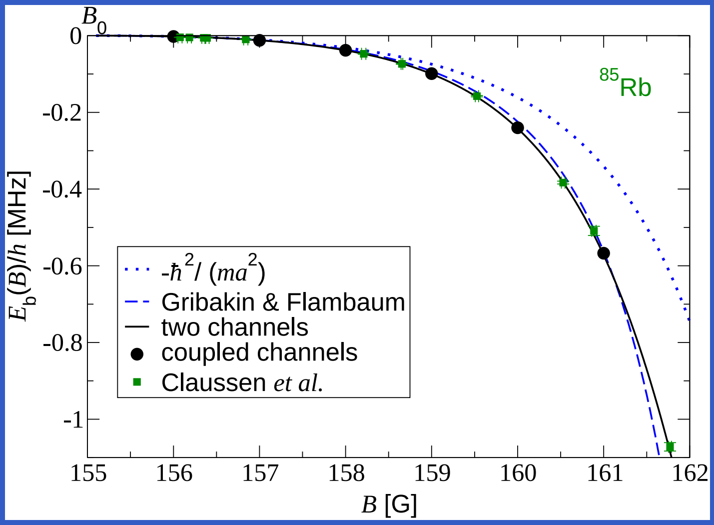

entrance-channel dominated

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$$\Updownarrow$$

$$\eta = \frac{\hbar^2/(m\bar{a}^2)}{a_\mathrm{bg}\mu_\mathrm{res}\Delta B/\bar{a}} = \frac{E_\mathrm{vdW}}{\Gamma_\mathrm{res}(E_\mathrm{vdW})} < 1$$

Classification of Feshbach resonances

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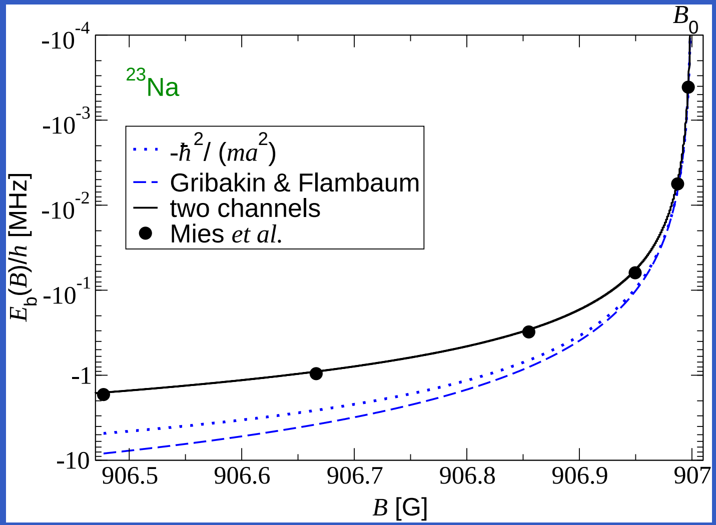

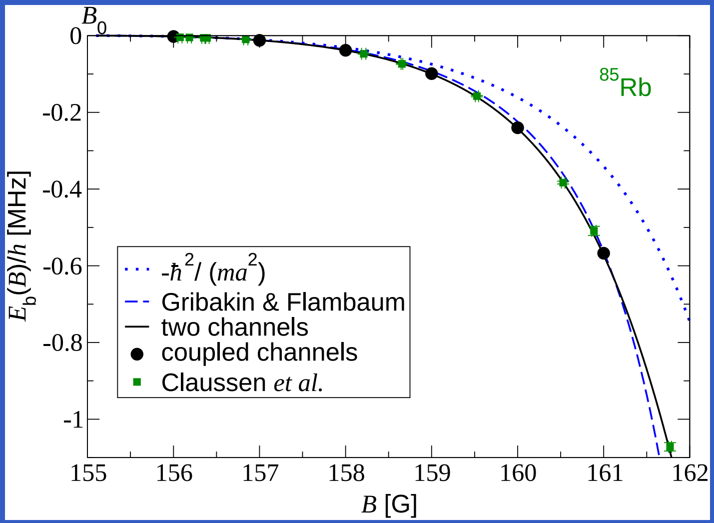

closed-channel dominated	$\eta > 1$	entrance-channel dominated	$\eta < 1$
^{6}Li 543 G	1215	^{40}K 202 G	0.46
^{23}Na 907 G	11	^{85}Rb 155 G	0.04
^{87}Rb 1007 G	5.9	^{6}Li 834 G	0.02

Review: TK, K. Góral, and P.S. Julienne, RMP **78**, 1311 (2006)

Diatomic Feshbach molecules

Two-component Feshbach-molecular bound states

- Schrödinger equation:

$$H_{\mathrm{2B}} \begin{pmatrix} \phi_{\mathrm{b}}^{\mathrm{bg}} \\ \phi_{\mathrm{b}}^{\mathrm{cl}} \end{pmatrix} = E_{\mathrm{b}} \begin{pmatrix} \phi_{\mathrm{b}}^{\mathrm{bg}} \\ \phi_{\mathrm{b}}^{\mathrm{cl}} \end{pmatrix}$$

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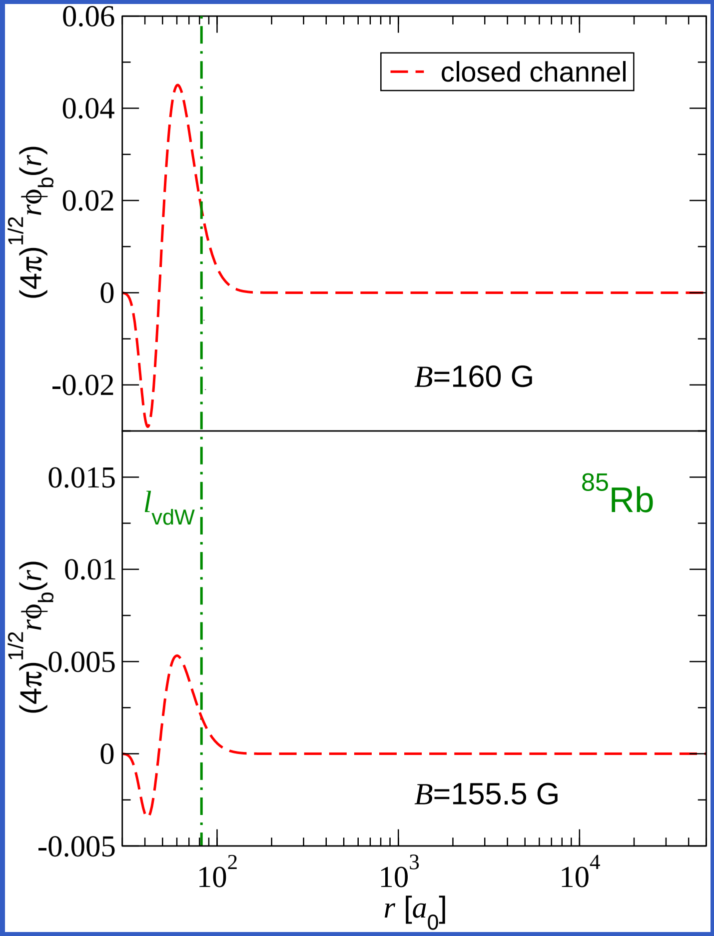

This two-channel approach: TK, T. Gasenzer, P.S. Julienne, and K. Burnett, PRL **91**, 230401 (2003)

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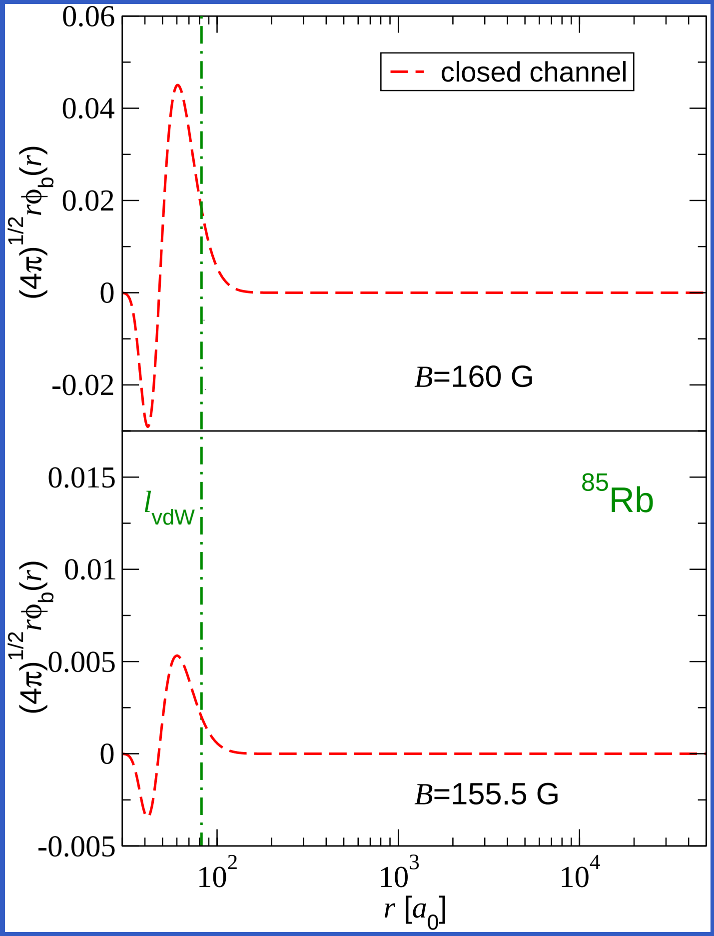
$$\mu_{\mathrm{b}}(B) = \frac{d}{dB} E_{\mathrm{b}}(B) = \mu_{\mathrm{res}} Z(B)$$



This two-channel approach: TK, T. Gasenzer, P.S. Julienne, and K. Burnett, PRL **91**, 230401 (2003)

Diatomic Feshbach molecules

Near-resonant Feshbach-molecules become universal!

- Schrödinger equation:

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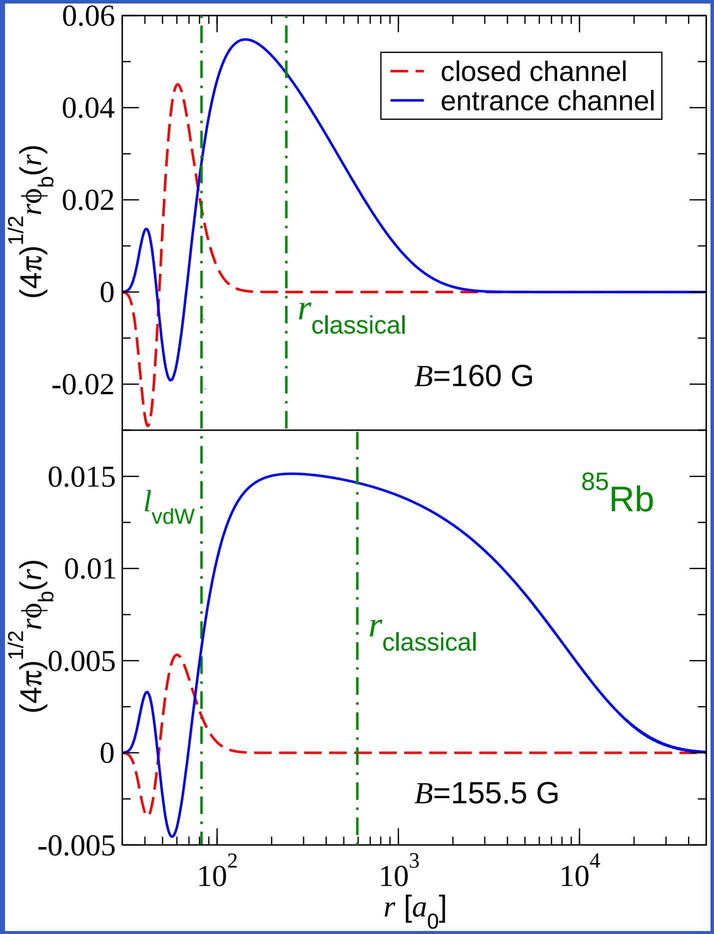
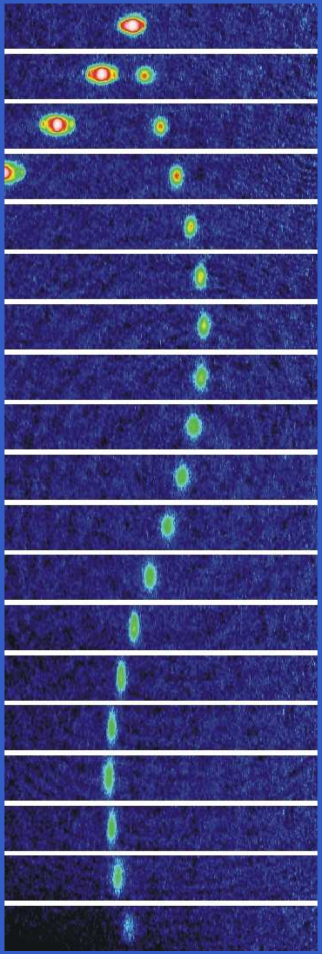


This two-channel approach: TK, T. Gasenzer, P.S. Julienne, and K. Burnett, PRL **91**, 230401 (2003)

Diatomic Feshbach molecules

Stern-Gerlach separation of ^{87}Rb atoms and molecules



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- Relative force:

$$\mathbf{F} = -\mu_{\mathrm{b}}(B) \boldsymbol{\nabla} B = -\mu_{\mathrm{res}} Z(B) \boldsymbol{\nabla} B$$

Experiment: S. Dürr, T. Volz, A. Marte, and G. Rempe, PRL **92**, 020406 (2004)
See also: J. Herbig *et al.*, Science **301**, 1510 (2003)

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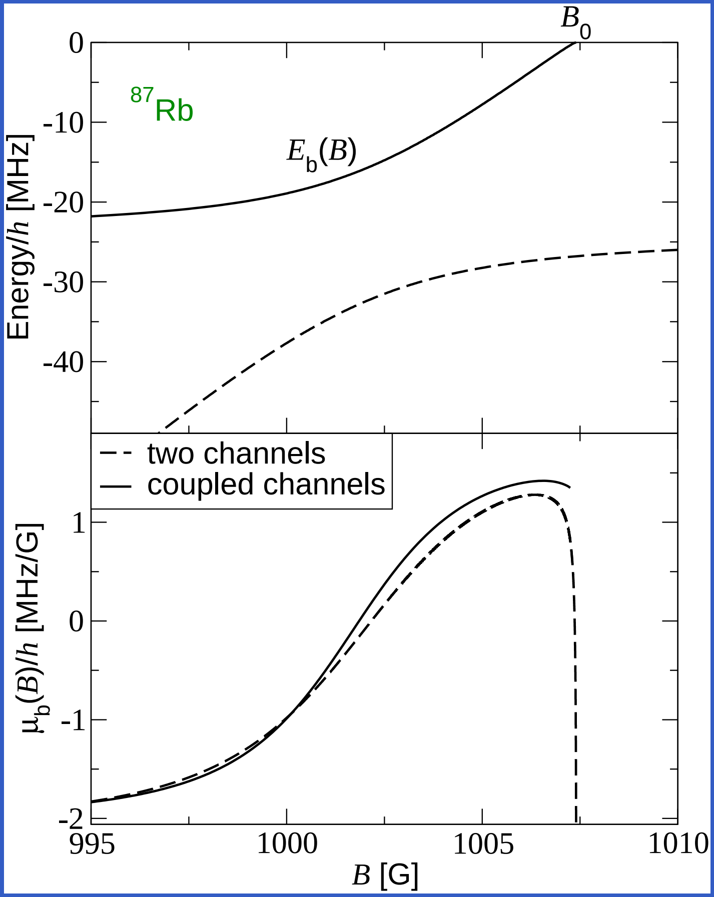

Coupled-channels theory: E.G.M van Kempen and B.J. Verhaar
This two-channel approach: K. Góral *et al.*, J. Phys. B **37**, 3457 (2004)

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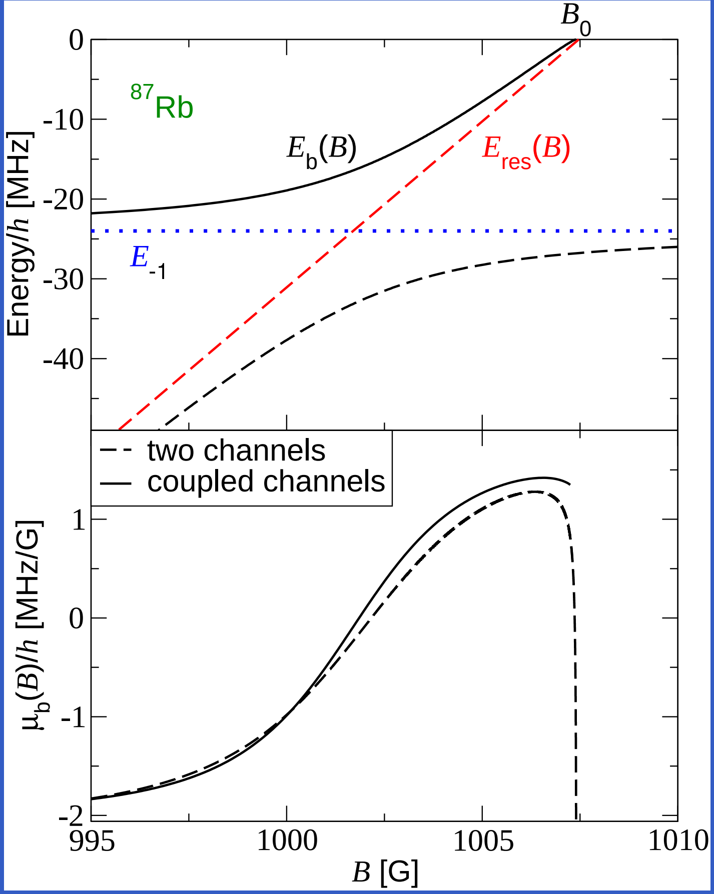

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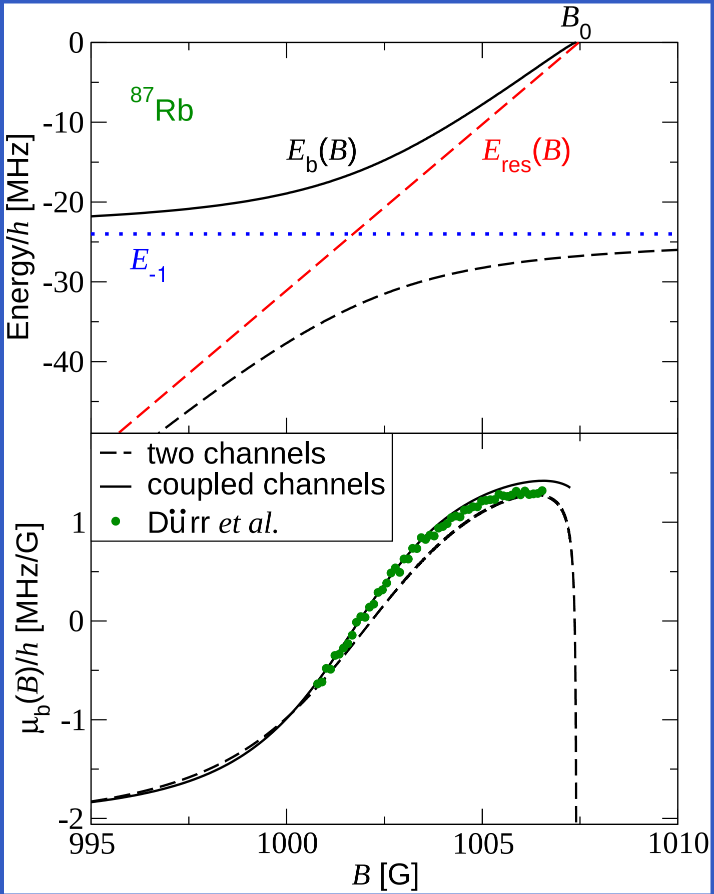

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Diatomic Feshbach molecules

Direct measurement of closed-channel admixtures

- Schrödinger equation:

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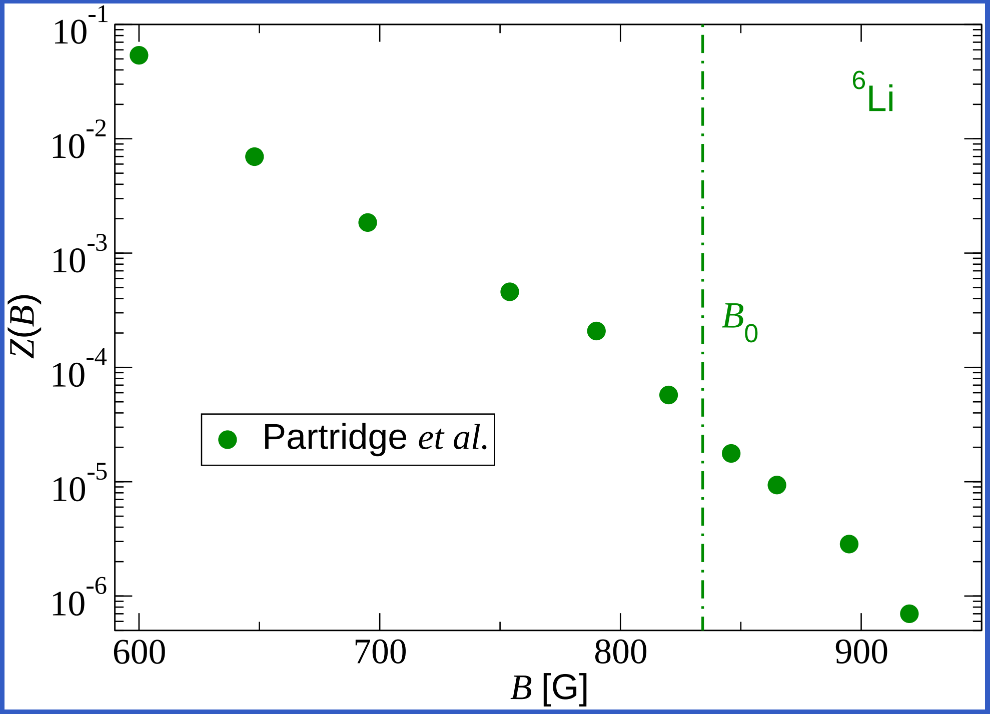
- Closed-channel admixture:

$$Z = \int d\mathbf{r} \left| \phi_{\mathrm{b}}^{\mathrm{cl}}(\mathbf{r}) \right|^2$$



Experiment: G.B. Partridge, K.E. Strecker, R.I. Kamar, M.W. Jack, and R.G. Hulet, PRL **95**, 020404 (2005)

Diatomic Feshbach molecules

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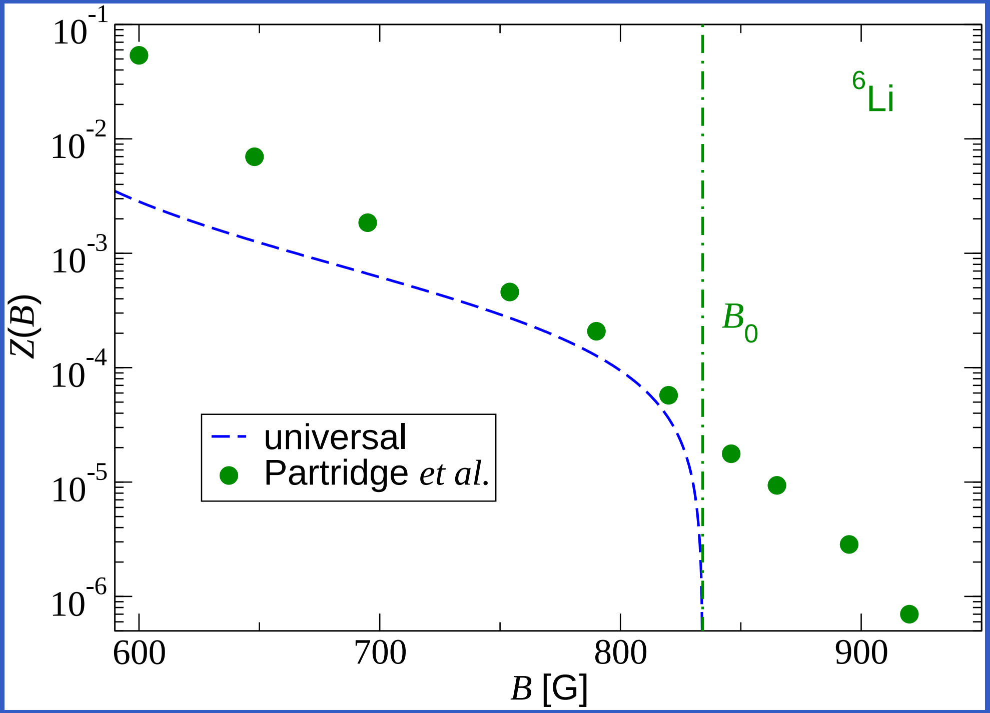

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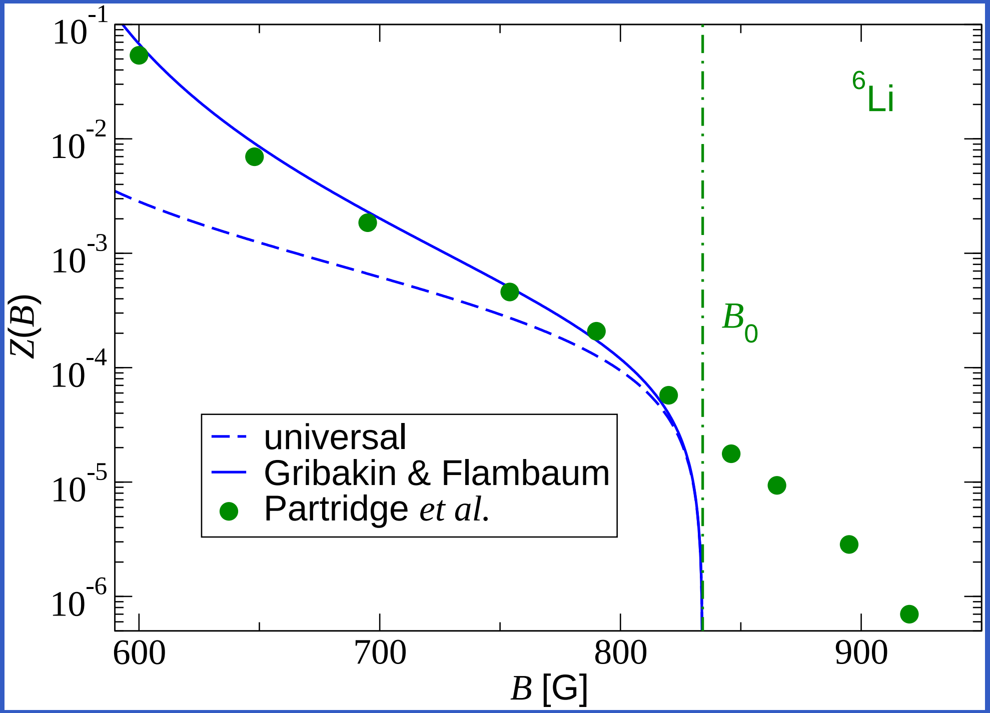

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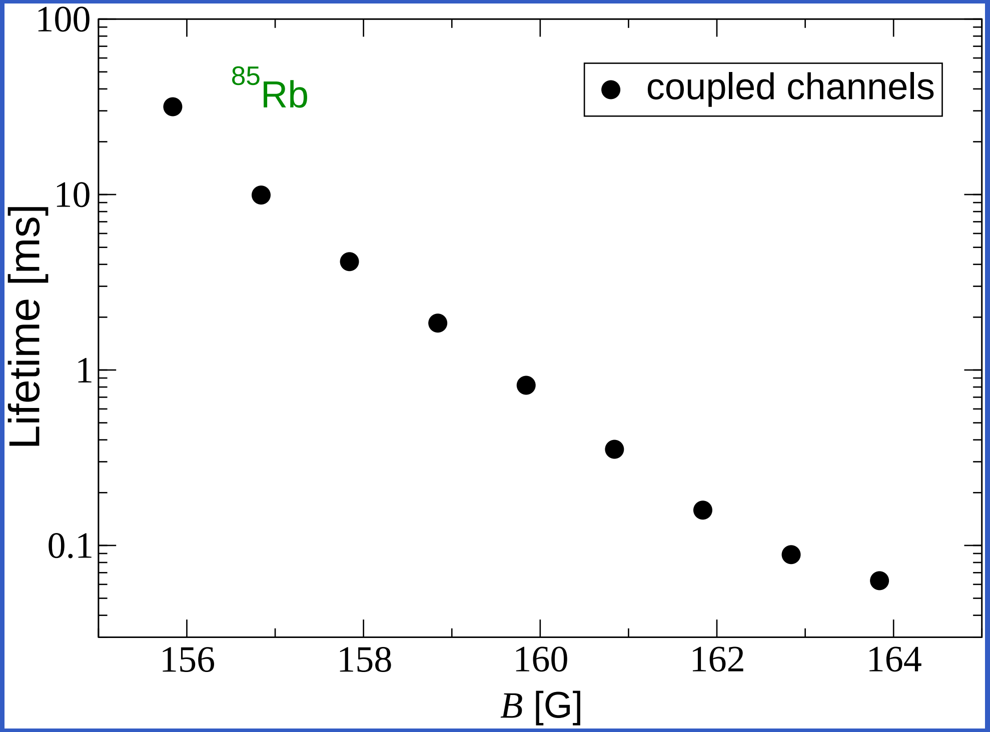
$\Updownarrow$

^{85}Rb$_2$ Feshbach molecules decay due to spin relaxation!

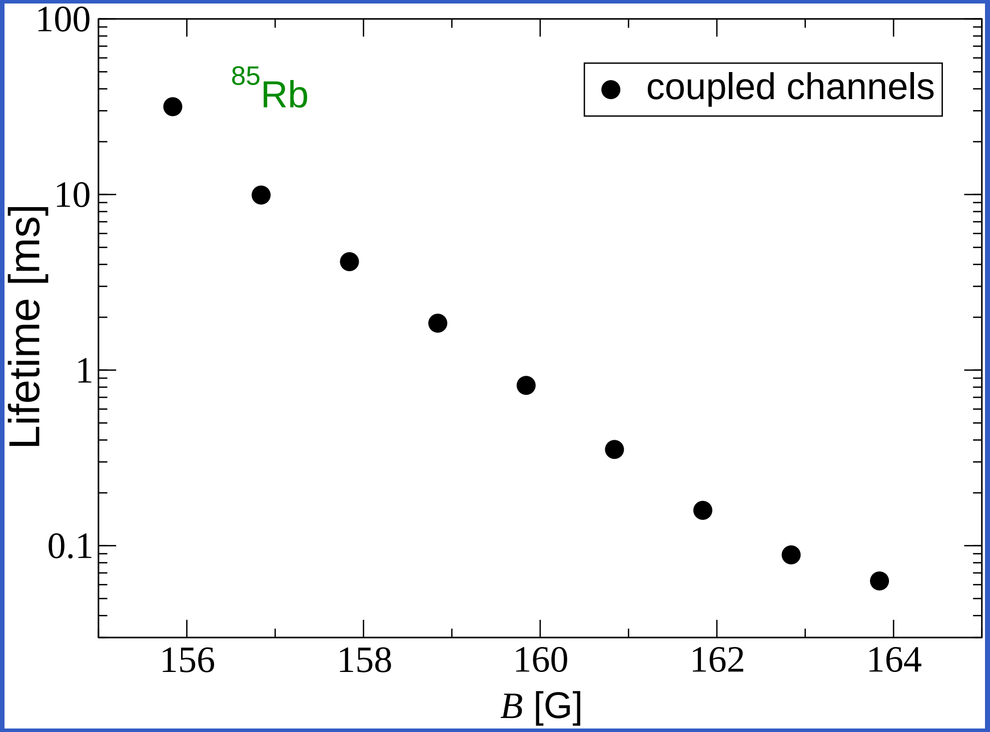


Diatomic Feshbach molecules

The lifetime depends on the size of the molecule!

- Average volume of a halo state:

$$\mathcal{V} = 4\pi\langle r^3\rangle/3 = \pi a^3$$



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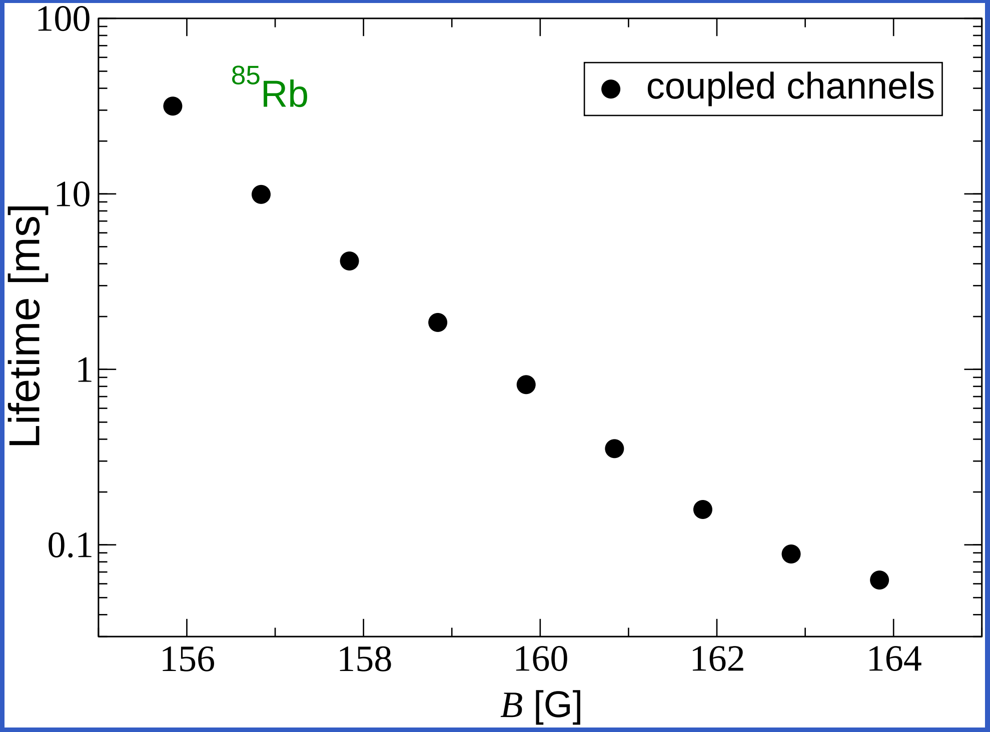

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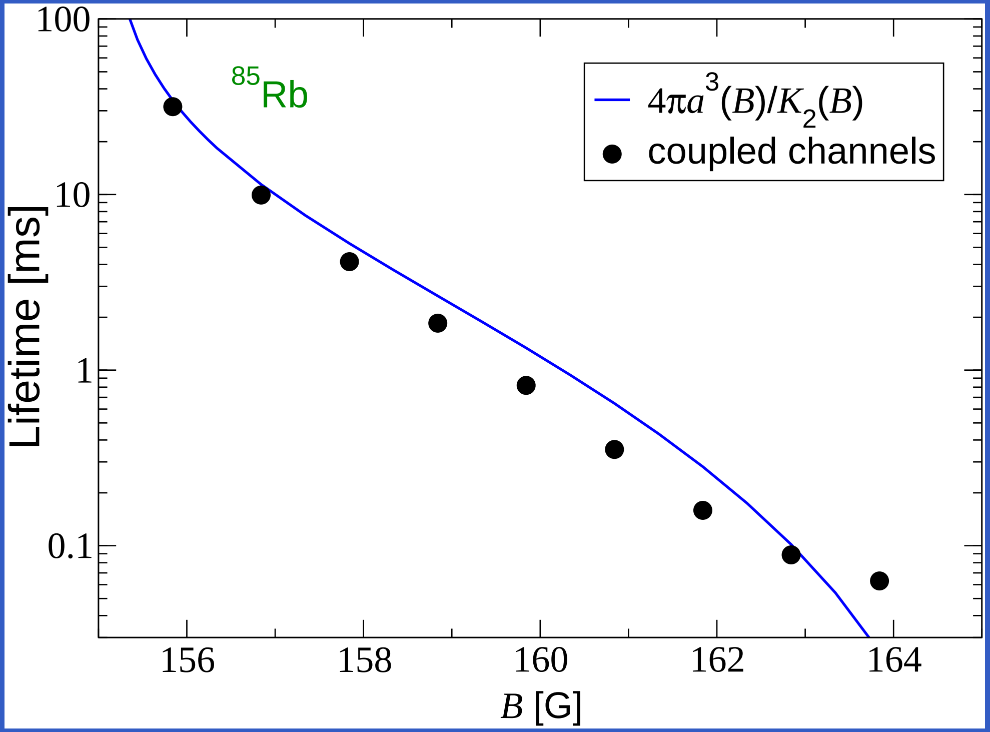

Diatomic Feshbach molecules

Experiment and theory agree!

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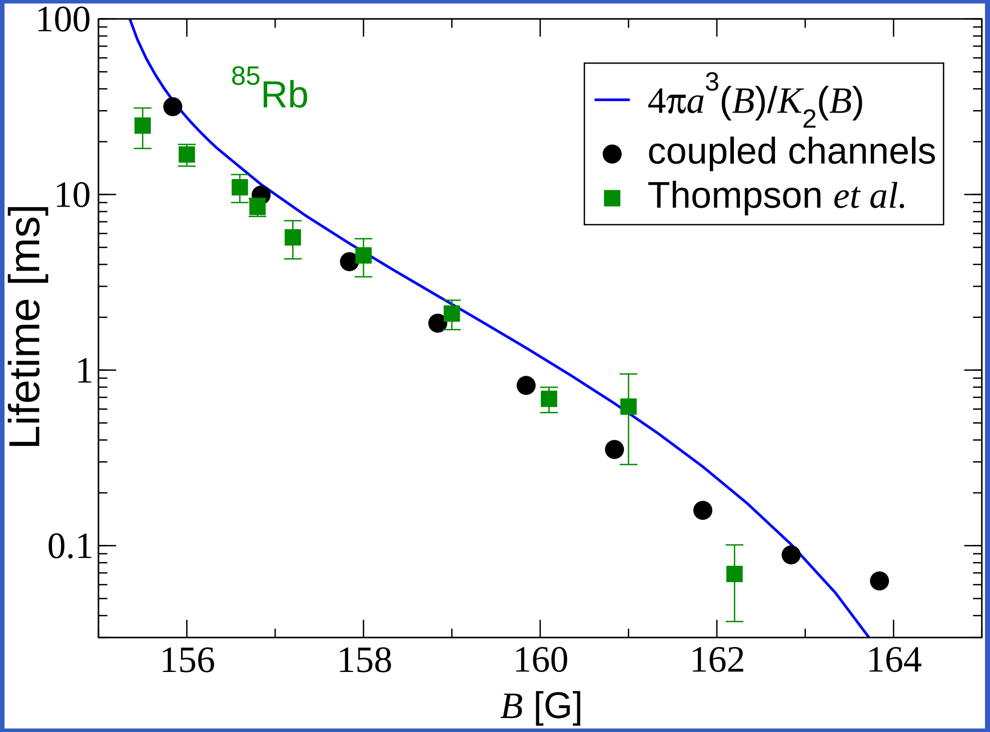

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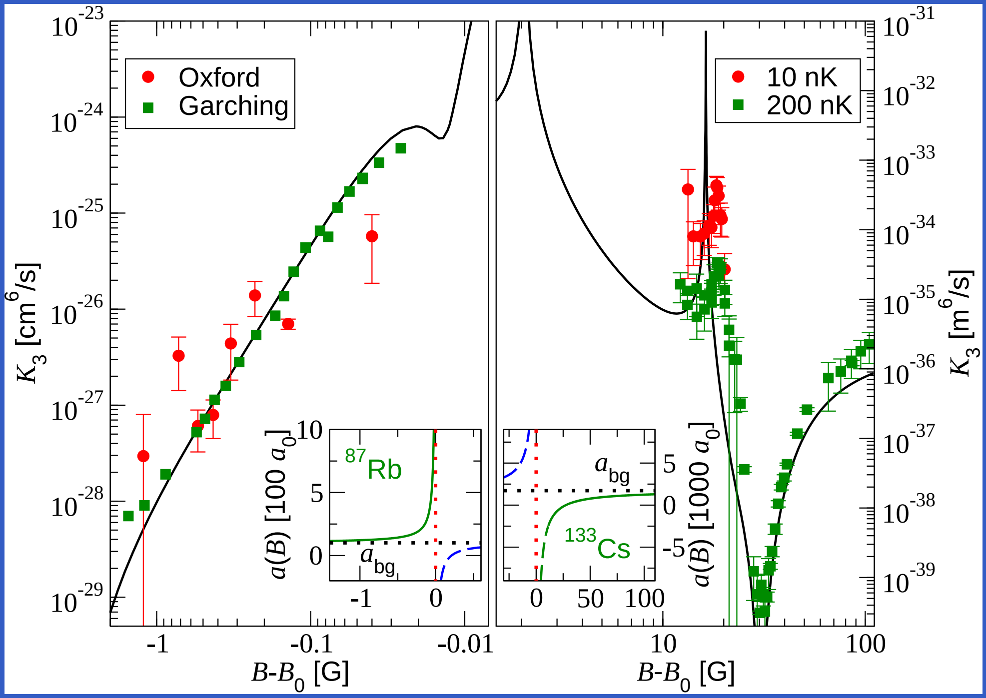
Theory: TK, E. Tiesinga, and P.S. Julienne, PRL **94**, 020402 (2005)

Outlook

- **Universal trimer molecules**



General concept: E. Nielsen and J.H. Macek, PRL **83**, 1566 (1999)
B.D. Esry, C.H. Greene, and J.P. Burke, PRL **83**, 1751 (1999)
Experiment: T. Kraemer *et al.*, Nature **440**, 315 (2006)
See also: G. Smirne *et al.*, PRA **75**, 020702(R) (2007)
J. Stenger *et al.*, PRL **82**, 2422 (1999)

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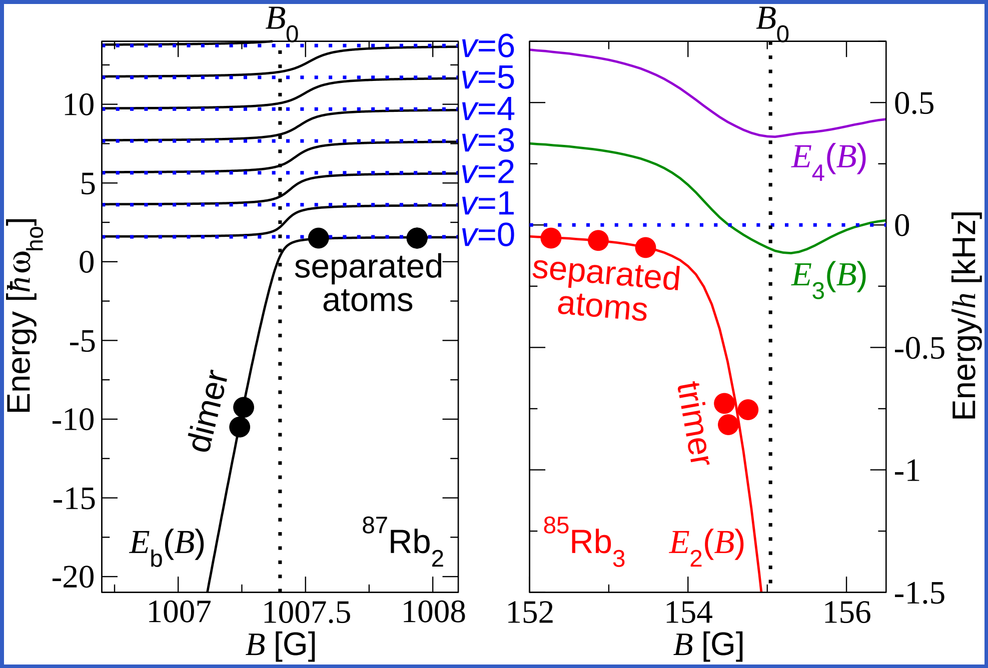
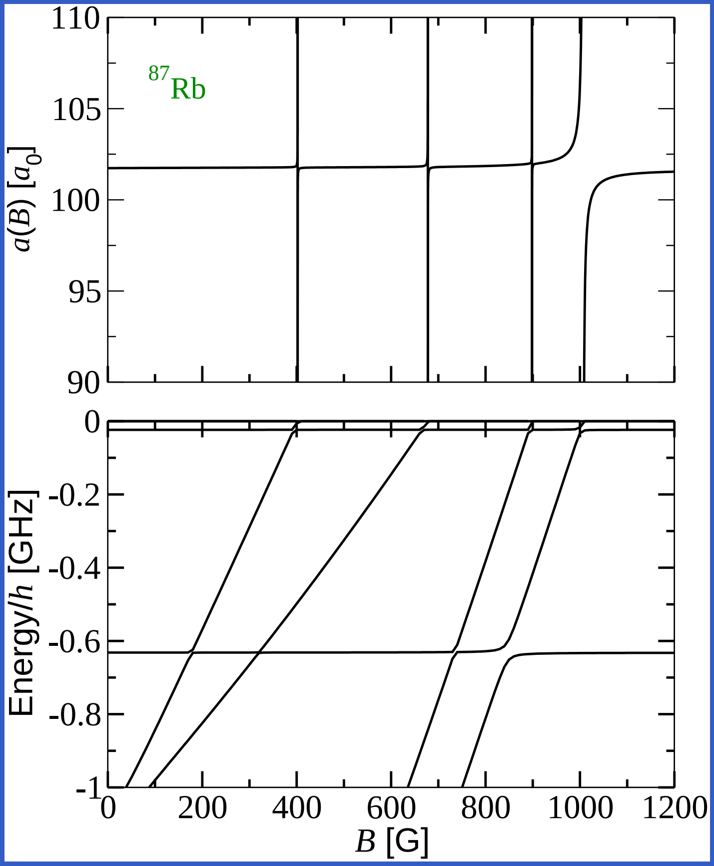


General idea: V. Efimov, Phys. Lett. **33B**, 563 (1970)
This concept: M. Stoll and TK, PRA **72**, 022714 (2005)
See also: V. Efimov, Nucl. Phys. A **362**, 45 (1981)
W. Schöllkopf and J.P. Toennies, Science **266**, 1345 (1994)

Outlook

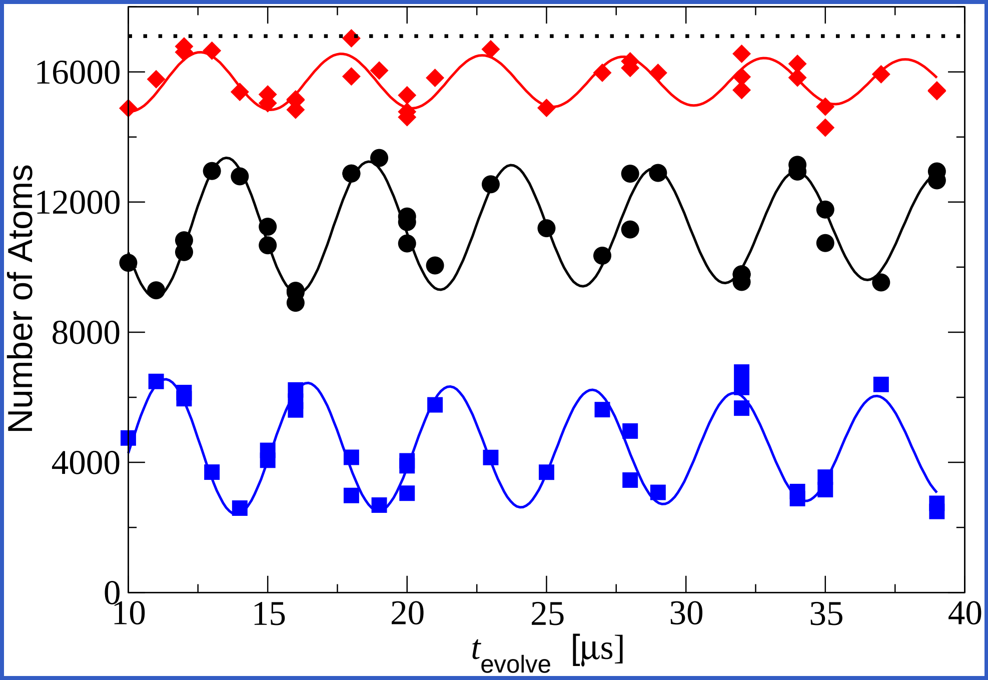
- Universal trimer molecules
- **Ground-state and polar molecules**



This concept: D. Jaksch *et al.*, PRL **89**, 040402 (2002)
This experiment: K. Winkler *et al.*, PRL **98**, 043201 (2007)
Figure: E. Tiesinga and P.S. Julienne

Outlook

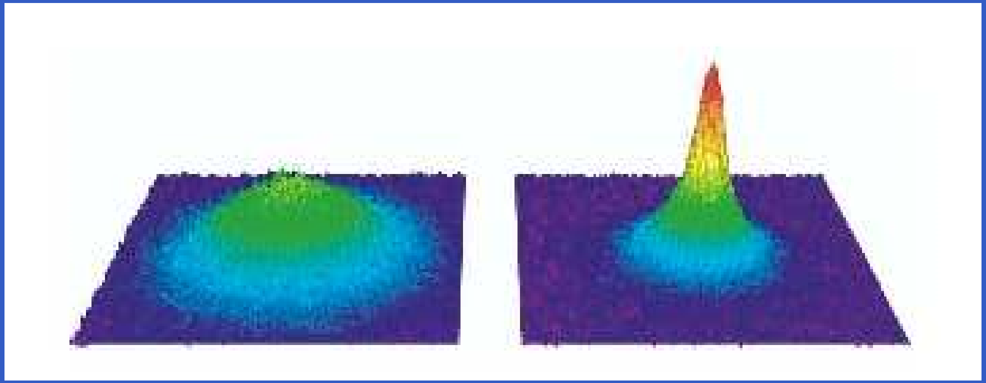
- Universal trimer molecules
- Ground-state and polar molecules
- **Dynamics of pairing and molecule formation**



Adapted from: E.A. Donley *et al.*, Nature **412**, 295 (2002)

Outlook

- Universal trimer molecules
- Ground-state and polar molecules
- Dynamics of pairing and molecule formation
- **Superfluid Fermi gases**



Adapted from: M. Greiner *et al.*, Nature **426**, 537 (2003)

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- Coupled-channels calculations: Paul Julienne (NIST)
- Experimental groups at: Oxford, JILA, Innsbruck, MIT,
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